Physics 411 Momentum Lecture

Momentum

We have had a glimpse of the great utility of the principle of conversation of energy when non-conservative forces are not present or do not perform work. Now we will see the utility of another fundamental conservation principle, that of the ***conservation of momentum***.

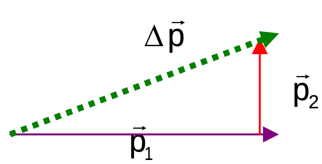
Momentum is a vector, the product of mass and velocity $\vec{p} \equiv m\vec{v}$.

When we use momentum to describe the winning streak of a team, that usage is close to the physics-al definition. An object with a lot of momentum is harder to curtail than one with less momentum. We will see how applying momentum conservation helps us in analyzing a variety of physical scenarios.

In general, a conservation principle tells us about the start and finish of a process. It says that the quantity at the start equals quantity at the end, or has a definite relation to the quantity at the end, independent of how one got from start to end. This is very useful in analyzing complex systems for which their evolution is intricate or nigh impossible to resolve in detail. However, we can speak with confidence of the over-all change in conserved quantities.

First an example that shows how to calculate Δp.

A car is going 12.0 m/s east. It then makes a left turn and proceeds 5.0 m/s north.Find the change in momentum, $\Delta\vec{p}$, (magnitude and direction relative to east).



We see that $|\Delta\vec{p}| = \sqrt{\vec{p}_1^{\,2} + \vec{p}_2^{\,2}} = 13$ m/s , the angle is given by noting that $\tan\theta = 5/12 \Rightarrow \theta = 22.6^o$ north of east (or 22.6^o ccw).

Momentum of two objects in an internal system (no net external forces).

What happens when two small objects moving relatively fast collide in outer space, far from any large gravitational field (huge massive body nearby).

Since the objects are small, their mutual attraction due to inertia may be ignored in comparison to their kinetic energies. Then upon collision,Newton's 3rd law reminds us that $\vec{F}_{12} = -\vec{F}_{21}$.

$m_1\vec{a}_{12} = -\,m_2\vec{a}_{21}$

If the collision takes place over a small time interval Δt, then $a \approx \Delta v/\Delta t$, so we may substitute:

$m_1 (\Delta\vec{v}_{12} / \Delta t) = - m_2 (\Delta\vec{v}_{21} / \Delta t)$

But $= m_1 \Delta\vec{v}_{12} = \Delta\vec{p}_1$ and similarly we obtain $\Delta\vec{p}_2$. We see then that in the absence of external forces, $\Delta\vec{p}_1 = - \Delta\vec{p}_2$

or the sum of the changes, $\Delta\vec{p}_1 + \Delta\vec{p}_2 = 0$ There is no net change in momentum, *momentum is conserved* in the absence of external forces.

This is true for any number of objects, not just two. In other words, say we are interested in the evolution of a system with N objects, that we are able to track and keep up wit their boundary at all times. The total momentum is

$\vec{p}_{tot} = \Sigma \vec{p}_i$

Σ means sum over all the individual momentums and *i* is an index running from 1 to N. Then if no net external forces act on the collective system,

$$\Delta\vec{p}_{tot} = \Sigma \Delta\vec{p}_i = 0$$

<u>Example</u>. A 2.0 g mass with velocity 6.5 cm/s to the left strikes a 3.0 g mass initially at rest. If the 2.0 g mass rebounds to the right at 2.5 cm/s, determine the final velocity (cm/s) of the 3.0 g mass. Assume no external forces.

Solution $\Delta\vec{p}_{tot} = 0$ means for two masses that $p_{1f} + p_{2f} – p_{1i} – p_{2i} = 0$ or rearranging:

$p_{1i} + p_{2i} = p_{if} + p_{2f}$ as $p = mv$, and assigning index 2 $\Leftrightarrow$ 3.0 g mass

Thus (2.0 g)(6.5 cm/s) + (3.0 g)(0) = (2.0 g)(-2.5 cm/s) + (3.0 g) (? cm/s)

$13 + 0 = -5 + p_{2f}$ $p_{2f} = 18$ g cm/s. so $v_{2f} = 18/3 =$ <u>6.0 cm/s to the left</u>

<u>Example</u>. A 28.0 g mass with velocity 3.0 m/s due east collides with a 52.0 g mass initially at rest. If the two stick together, find their final velocity.

Here, $p_{1i} + p_{2i} = p_{(i+2)f}$, so (28.0 g)(3.0 cm/s) + 0 = ((28 +52)g) (v_f)

(v_f is obviously directed due east)

Final speed is (28 x 3)/70 = **1.2 m/s**

The above case is an example of a *perfectly inelastic* collision. In general, in an inelastic collision, energy is not conserved (some energy went to “sticking” the objects together), but momentum IS nevertheless conserved.

Impulse. Sometimes we are interested in the action of a force that acts in a short time interval, an impulsive force. We might be interested in a design of a specific component of a vehicle, or of a trampoline that softens the impact of collision.
It is useful to define a quantity known as the *impulse*, which is the product of the force over a small time interval.

$$\textbf{impulse} \equiv \vec{F}\Delta t.$$

But from arguments similar to just above that this latter quantity is just $\Delta\vec{p}$.

Thus we arrive at the impulse momentum theorem: $\Delta\vec{p} = \vec{F}\Delta t$

This relation is strictly true when the impulsive force is constant over the time interval considered. Otherwise, we must interpret $\vec{F}$ as the <u>average</u> force over that interval.

Example. A car moving at 20 m/s to the east crashes into a tree. Estimate the force acting on a passenger of mass 52 kg according to the following scenarios:

(a) no seatbelt. Passenger is brought to a halt by the dashboard in 3.00 ms.
(b) seat belt. Passenger is brought to a halt in 0.3 s
(c) air bag. Passenger is brought to halt in 1.0 second.

Solution: $\Delta\vec{p} = \vec{F}_{imp}\Delta t$ so $\vec{F}_{imp} = \dfrac{\Delta\vec{p}}{\Delta t}$ in each case. Also, the momentum change

$\Delta\vec{p} = 0 - (52\text{ kg})(20\text{ m/s}) = -1040\text{ kg.m/s}$ to the east is the same.

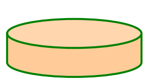
(a) $\vec{F}_{imp} = -1040/0.003\text{ s} = -3.47 \times 10^5\text{ N}$

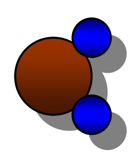
(b) $\vec{F}_{imp} = -1040/0.3\text{ s} = -3.47 \times 10^3\text{ N}$

(c) $\vec{F}_{imp} = -1040/1.0\text{ s} = -1.04 \times 10^3\text{ N}$

Center of Mass

So far we've primarily concerned ourselves with describing the motion of a point particle or two. What if our system encompasses many such point particles. Or suppose we wish to describe the motion of particle with internal structure or shape?

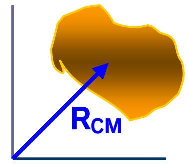


(Or see swarm of particles on page 1 of Energy lecture)

Answer: Collection of particles can be described by a special point—a vector when referenced in a particular coordinate frame—that is related to the mass rated average of the position of all the points called the center of mass (**CM**), The definition of this special vector, $\mathbf{R}_{CM}$, shows us how to determine it:

$$\mathbf{R}_{CM} = \frac{1}{M}\sum_{i=1}^{N} m_i \vec{r}_i$$



where M is the total mass and $\mathbf{r}_i$ gives the vector position of the ith particle. The components of $\mathbf{R}_{CM}$ can be found separately, of course.

Example. Determine the coordinates of the center of mass of the following three points,

1: 1.5 g (6 cm, -2 cm) **2**: 6.0 g (-3 cm, 1 cm) and **3**: 4.5 g (0 cm, 4 cm)

Solution: Common sense tells us that in 2 dimensions, the center of mass be bounded by the triangle formed by joining these three points, We proceed by finding the total mass M = (1.5 + 6.0 + 4.5) g = 12 g

Then $\mathbf{X}_{CM}$ = (1/(12 g))((1.5 g)*6 + (6 g)*(-3) + (4.5 g)*(0)) = -0.75 cm

$\mathbf{Y}_{CM}$ = (1/(12 g))((1.5 g)*(-2) + (6 g)*(1) + (4.5 g)*(4)) = +1.75 cm

So $\mathbf{R}_{CM}$ = ($\mathbf{X}_{CM}$, $\mathbf{Y}_{CM}$) = **(<u>-0.75 cm, 1.75cm</u>)**

Total momentum and center of mass. If we differentiate the expression for $\mathbf{R}_{CM}$, that's just the calculus way of finding the rate of change of the position vectors with time, which is the velocities—we get: $\mathbf{V}_{CM} = \frac{1}{M}\sum_{i=1}^{N} m_i \vec{v}_i$

Multiply both sides by the total mass M and we obtain:

$$M\mathbf{V}_{CM} = \sum_{i=1}^{N} m_i \vec{v}_i$$

but this sum is just the *total momentum*, $\mathbf{P}_{tot}$

Another significance of the center of mass, the center of mass velocity times the total mass gives the total momentum,

$$\mathbf{P}_{tot} = M\mathbf{V}_{CM}$$

This means that we can treat the collection of many particles as *one* point particle with mass M and velocity $\mathbf{V}_{CM}$.

Acceleration and center of mass. If we differentiate the center of mass velocity, we arrive at the center of mass acceleration,

$d\mathbf{P}/dt = M\mathbf{A}_{CM}$ which must by Newton's 2nd law, F_{net}.

But as the right side has the dimensions of a force, so must the left side, viz, $F_{ext} = d\mathbf{P}/dt$

This would be a trivial observation, except that we notice that, as momentum is the product of mass and velocity, by the chain rule (for finding differences)

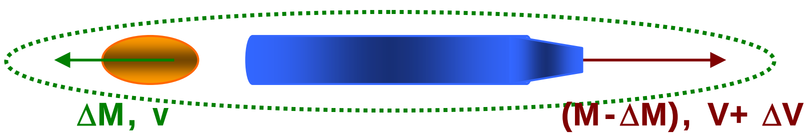
$$\Delta\mathbf{p} = \Delta(m\mathbf{v}) = m(\Delta\mathbf{v}) + \mathbf{v}(\Delta m)$$

(obviously, dividing by (Δt doesn't affect this))

The implication is that by expressing force equal to the rate of change of momentum with time, instead of just mass times acceleration***, we allow for momentum to change by a change in mass with time***. This is precisely the principle behind rocket **thrust**.

Consider a system, initial M moving with speed V.

Let an amount of mass be ejected from the object at rate $R = \Delta M/\Delta t$ and with speed v. Thus, in time interval ΔT, the original object has mass $M - \Delta M$. Denote its new speed $V' = V + \Delta V$



Since the change was incurred internally, there was no EXTERNAL force. Notice how the boundary includes both objects. Thus momentum has to be conserved:

$P_{initial} = P_{final}$ or $MV = \Delta M(-v) + (M-\Delta M)(V+\Delta V)$ velocity to the right being (+).
$-(\Delta M)(V + v + \Delta V) + M\Delta V = 0$ or $(\Delta M)(V + v + \Delta V) = M\Delta V$.

But $V+v+\Delta V$ is just the relative speed of the lost mass from the rocket's point of view: relative speed between 2 frames equals $| v_2 - v_1 |$ Thus in our case $V+\Delta V - (-v) = V + v + \Delta V$. This cumbersome term is called the *exhaust velocity,* u. Simplifying, $(\Delta M)(V + v + \Delta V) = \Delta M u = M\Delta V$. Dividing both sides by Δt,

$(\Delta M/\Delta t)u = M[\Delta V/\Delta t]$ or using definition of rate of mass loss and Newton's 2nd law: $Ru = Ma_{rocket}$ The product Ru is called the *thrust* of the rocket. For short time intervals the acceleration of the rocket is given by $a_{rocket} = Ru/M$.

Example. Rocket mass 850 kg, ejects mass R = 2.3 kg/s. u = 2800 m/s. Find thrust of rocket engine and initial acceleration T = 6400 N, a = 7.6 m/s^2

Collision- we will consider an event a collision when two or more objects bump into each other and the result of the mutual impulsive interactions changes their direction or speeds in some way. Usually the forces acting during the collision are hard to track and analysis of these forces, while do-able, is complex and intricate. It's easier to observe before and after the collision takes place. In our collisional model, we assume that we can clearly distinguish between before collision and after collision.

There are two types of (macroscopic, mechanical) collisions:

Elastic collisions- total kinetic energy is the same before and after, as well as momentum conserved

Inelastic collision- total kinetic energy is not the same (in fact less), but momentum still conserved. Some of the initial kinetic energy was used in partially deforming or heating the objects. If the two object stick completely together, the event is termed is a *perfectly inelastic collision*.

Depending on whether collision is elastic or not, we solve the problem differently. If the problem does not state elastic, assume inelastic. Now, for 2-D collisions:

1. draw before and after diagrams
2. organize info of m's , v's and angles
3. resolve into x and y components notice all the subscripts for which mass, before or after, x or y:

$$m_1v_{1ix} + m_2v_{2ix} = m_1v_{1fx} + m_2v_{2fx}$$
$$m_1v_{1iy} + m_2v_{2iy} = m_1v_{1fy} + m_2v_{2fy}$$

4. if collision are inelastic only momentum conserved

   if collision are perfectly inelastic , 1 and 2 move together after collision, so $v_{1fx} = v_{2fx}$ and $v_{1fy} = v_{2fy}$

5. if collision is elastic total kinetic energy conserved as well as momentum (the work is a lot simpler) $\frac{1}{2} m_1v_{1i}^2 + \frac{1}{2} m_2v_2^2 = \frac{1}{2} m_1v_{1f}^2 + \frac{1}{2} m_2v_{2f}^2$
6. If one mass strikes a stationary mass and only the relative angle ϕ between departing masses is known ($\phi = \theta_1 + \theta_2$), then a useful formula to find that angle is :

$$\mathbf{p_i^2 = p_{1f}^2 + p_{2f}^2 + 2p_{1f}p_{2f}\cos\phi}$$

This formula holds true for all collisions for which $F_{ext} = 0$.

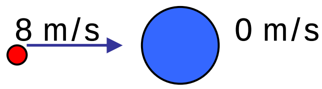
7. Solve for unknown quantities

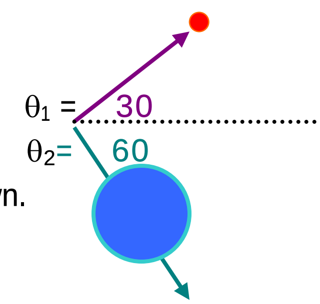
We've already done two examples of collisions in 1 dimension. Now, let's take on 2 dimensions:

<u>Example.</u> On an air table (which greatly reduces friction) a small puck 0.40 kg (8.0 m/s to east) collides with larger one (0.50 kg) initially at rest. After collision, the small puck leaves at angle 30^o north of east and larger one at angle 60^o south of east.

(a) Find the final speeds of each puck.

(b) Is the collision elastic? If not, find the energy lost (magnitude only).

before 

after: 

Seek speeds v_{1f} and v_{2f} as the directions are already known.

Organize information:

$m_1 = 0.4$ kg $v_{1i} = 8.0$ m/s east $\theta_{1i} = 0^o$ $m_2 = 0.5$ kg $v_{2i} =$ 0 m/s

$v_{1f} = ?$ m/s $\theta_1 = 30^o$ N of E $v_{2f} = ?$ m/s $\theta_2 = 60^o$ S of E

First conserve momentum:

$m_1v_{1ix} + m_2v_{2ix} = m_1v_{1fx} + m_2v_{2fx}$

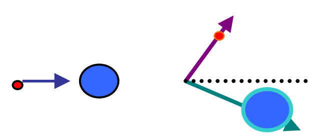
$m_1v_{1iy} + m_2v_{2iy} = m_1v_{1fy} + m_2v_{2fy}$

In terms of geometry of problem:

$m_1(v_{1i}\cos(0)) + m_2(0) = m_1v_{1f}\cos(30) + m_2v_{2f}\cos(60)$

$m_1(v_{1i}\sin(0)) + m_2(0) = m_1v_{1f}\sin(30) - m_2v_{2f}\sin(60)$

Since $m_2/m_1 = 1.25$, we have:



$(\cos(0)) = m_1v_{1i} = m_1v_{1f}\cos(30) + (1.25m_1)v_{2f}\cos(60)$ or dividing by m_1

$v_{1ix} = \cos(30)\ v_{1f} + 1.25\cos(60)\ v_{2f}$

$8\text{ m/s} = 0.866v_{1f} + 1.25\cos(60)\ v_{2f} \Rightarrow 0.866v_{1f} + 0.625v_{2f} = 8$

For y direction: $m_1v_{1i}(\sin(0)) + m_2(0) = m_1v_{1f}\sin(30) - m_2v_{2f}\sin(60)$ (why the minus sign?)

cut to the chase, there's NO y component initially!! So :

$$\text{ZERO} = 0 = m_1v_{1f}\sin(30) - m_2v_{2f}\sin(60)$$

$$= m_1v_{1f}\sin(30) - [1.25m_1]v_{2f}\sin(60)$$

Eliminating m_1,and substituting values for trig functions:

result of y analysis: $0.5v_{1f} - 1.0825v_{2f} = 0$

result of x analysis: $0.866v_{1f} + 0.625v_{2f} = 8.0$ m/s

Use the 1st one, y-analysis, to get $v_{1f} = 2.165v_{2f}$ thus substitution into x-analysis

$0.866(2.165)\,v_{2f} + 0.625v_{2f} = 8.0$ m/s or $[1.875 + 0.625]v_{1f} = 8$

$2.500v_{2f} = 8$ $v_{2f} =$ **3.200 m/s** 60^o S of E

Now substitute v_{2f} into y-analysis (it's a little simpler)

$0.5v_{1f} - 1.0825\times(3.20) = 0$ we obtain $\mathbf{v_{1f}}$ **= 6.928 m/s** N of E to 4 sig figs.

(b) Kinetic energy analysis

$K_{initial} = ½\, m_1v_{1i}^2 + ½\, m_2v_2^2$ $K_{final} = ½\, m_1v_{1f}^2 + ½\, m_2v_{2f}^2$

$½(0.40\text{ kg})(8.0\text{ m/s})^2 + 0 = 12.8$ J $½(0.50\text{ kg})(3.2\text{ m/s})^2 + ½(0.40)(6.9\text{ m/s})^2 = 12.1$ J

2.56 + 9.60

Since $K_f \neq K_i$, collision is inelastic. (slightly) $\Delta K = 12.2 - 12.8 = -0.6$ J . So kinetic energy loss is 0.6 J

Check. Momenta must be conserved. When $\phi = \theta_1 + \theta_2 = 90$, it's easy to check: As the sum of the angle between the final momenta is 90, since $\mathbf{p_{1i}} = \mathbf{p_{1f}} + \mathbf{p_{2f}}$ (vector) take dot product:

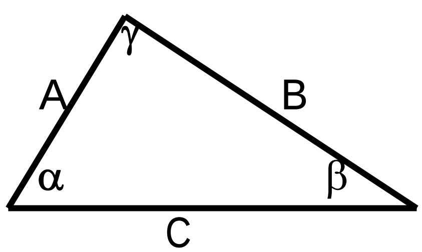
$p_{1i}^2 = p_{1f}^2 + p_{2f}^2$ (cross term $2p_{1f}p_{2f}\cos\phi$ vanishes). Using scaled mass

$(8)^2 = (1.25)^2(3.2)^2 + (6.928)^2$ checking $64 \approx 16 + 47.99$

See the above trig gymnastics spelled out in 2-body elastic collision discussion below.

Let us do the same calculation: On an air table (which greatly reduces friction) a small puck 0.40 kg (8.0 m/s to east) collides with larger one (0.50 kg) initially at rest. After collision, the small puck leaves at angle 30^o north of east and larger one at angle 60^o south of east. (a) Find the final speeds of each puck.

—But employ trigonometry much more sagaciously:



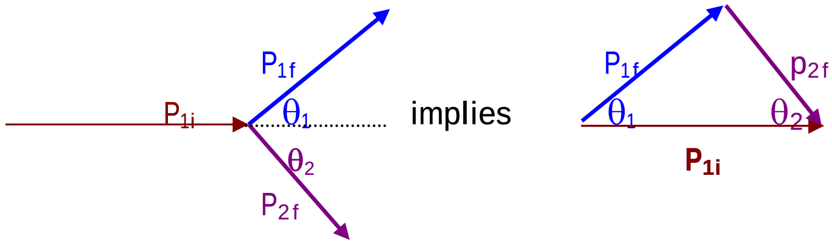
Recall the sine law:

$$\frac{C}{\sin\gamma} = \frac{A}{\sin\beta} = \frac{B}{\sin\alpha}$$

where $\gamma = 180 - (\alpha+\beta)$

We will use this construction to solve for the angles alpha and beta.

Momentum conserved means that $p_{1i} = p_{1f} + p_{2f}$ always obeys a triangular relation:



Thus by the sine rule, with the base C ≡ the initial momentum, p_{1i}:

$$\frac{p_{1i}}{\sin(180-[\theta_1+\theta_2])} = \frac{p_{1f}}{\sin\theta_2} = \frac{p_{2f}}{\sin\theta_1}$$

In our case, $\theta_1 + \theta_2 = 90$, so that $p_{1i}\sin\theta_2 = \mathbf{p_{1f}}$ and $p_{1i}\sin\theta_1 = \mathbf{p_{2f}}$

Substituting: (3.2 kg·m/s) sin(60°) = $\mathbf{p_{1f}}$ = 2.77 kg m/s mass = 0.4 kg

and (3.2 kg·m/s) sin(30°) = $\mathbf{p_{2f}}$ = 1.60 kg m/s mass = 0.5 kg

Thus $\mathbf{v_{1f}}$ = 2.77/0.4 kg = 6.93 m/s 30° N of E Similarly, $\mathbf{v_{2f}}$ = 1.6/0.5 = 3.2 m/s 60° S of E

This technique works whether or not the angles add to 90°.

Ex. In the absence of external forces, a puck 0.80 kg (5.0 m/s to east) collides with another one (0.50 kg) initially at rest. After collision, the small puck leaves at angle 25° north of east and larger one at angle 45° south of east. Determine the final speeds of each puck (m/s).

Solution: "1": p_{1i} = 4.0 kg m/s to the East. p_{1f} = ? m/s 45° S of E ; p_{2f} = ? m/s 25° Nof E

We seek p_{1f} and p_{2f}, from which we will obtain speeds by dividing by the respective masses. Next, the third angle opposite the initial momentum γ = 180 – 70 = 110°, thus here,

$$\frac{p_{1i}}{\sin(110)} = \frac{p_{1f}}{\sin 25} = \frac{p_{2f}}{\sin 45}.$$ Thus,

$p_{1f} = \dfrac{\sin(25)\cdot p_{1i}}{\sin(110)}$ = **<u>1.80 kg·m/s</u>** and so v_{1f} = (1.8 kg·m/s) ÷ 0.8 = <u>2.25 m/s 45° S of E</u>

$p_{2f} = \dfrac{\sin(45)\cdot p_{1i}}{\sin(110)}$ = **<u>3.080 kg·m/s</u>** and so v_{1f} = (3.00 kg·m/s) ÷ 0.5 = <u>6.00 m/s 25° N of E</u>

Which method do you prefer ?

<u>APPENDICES</u>

Elastic Collision, masses equal. It can be shown that for an elastic collision between objects with equal masses the only geometry other than 0° or 180° that can be realized by the struck masses is relative angle $\phi = 90° = \theta_1 + \theta_2$

Proof: Assume the 2nd mass is initially stationary. Energy is conserved so:

$$\tfrac{1}{2} m_1 v_{1i}^2 = \tfrac{1}{2} m_1 v_{1f}^2 + \tfrac{1}{2} m_2 v_{2f}^2$$

As mass m is the same, energy conservation statement simplifies to (multiply by 2):

$$v_{1i}^2 = v_{1f}^2 + v_{2f}^2$$

(Note: The above expression has that "right triangle length relationship" structure)
Momentum is conserved so vector $\mathbf{p_{1i}} = \mathbf{p_{1f}} + \mathbf{p_{2f}}$ Dot product of each side gives:

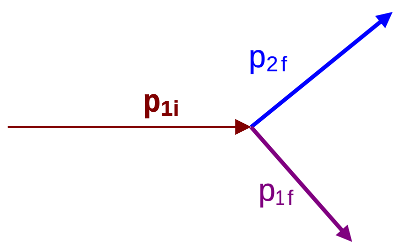
$$p_{1i}^2 = p_{1f}^2 + p_{2f}^2 + 2p_{1f}p_{2f}\cos\phi$$

where $p_{1i} = |\,\mathbf{p_{1i}}\,|$, the magnitude of $\mathbf{p_{1i}}$, and similarly for the other momenta.

Divide p_{1i}^2, p_{2f}^2 and $p_{1f}p_{2f}$ by m, $p_{1i}^2/m = m_1v_{1i}^2$ = twice KE , etc.
Thus, as $m_1 = m_2 = m$,

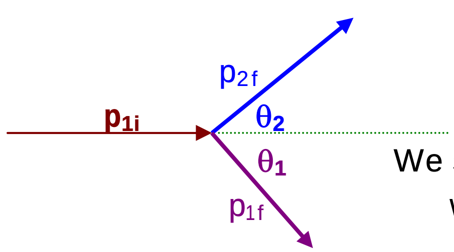
$$mv_{1i}^2 = mv_{1f}^2 + mv_{2f}^2 + 2m\, v_{1f}v_{2f}\cos\phi \qquad \text{dividing out m's,}$$

momentum relation gives: $v_{1i}^2 = v_{1f}^2 + v_{2f}^2 + 2v_{1f}v_{2f}\cos\phi$ But it is also true that
energy relation gives: $v_{1i}^2 = v_{1f}^2 + v_{2f}^2$
This can only occur if third term vanishes ∴ $\cos\phi = 0$ or $\phi = \theta_1 + \theta_2 = 90^o$—the relative angle of the departing struck masses is mutually perpendicular!!

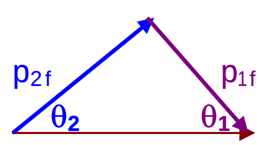


Example. A puck of mass m going 15.6 m/s strikes another stationary puck of mass m. *The collision is elastic*. One puck veers off at 14.4 m/s, the other at 6.0 m/s. Find the angles θ_1 and θ_2. Assume no friction.

Solution: As collision is elastic, $\theta_1 + \theta_2 = 90^o$. Another way of seeing the vectorial relation between p_{1i}, p_{1f} , and p_{2f} is to arrange them as a right triangle:



We see that $\mathbf{p_{2f}} = \mathbf{p_{1i}} \cos\theta_2$
whereas $\mathbf{p_{1f}} = \mathbf{p_{1i}} \cos\theta_1$



Let "1" correspond to mass that leaves at 6.0 m/s. Then $\theta_1 = \cos^{-1}(p_{1f}/p_{1i})$
$= \cos^{-1}(6.0/15.6) = 67^o$ and $\theta_2 = 90 - \theta_1 = 23^o$

Elastic Collision; masses not equal In this unwieldy case, it's much easier to find the relative angle ϕ between the struck masses than the individual angle and we shall restrict ourselves to this simpler case.

Example: An object of mass 4.000 g travelling 5.000 cm/s to the right collides with a stationary object (mass = 5.760 g). The collision is elastic. Find the angle ϕ between the struck object and the lighter object if the lighter one departs with speed 4.00 cm/s and the heavier one departs with speed 2.500 cm/s.

Solution:

The collision is indeed elastic, for. computing kinetic energies before and after,

Initial **Final**

$½4.0\cdot(5\text{ cm/s})^2 = ½4\cdot(4.0\text{ cm/s})^2 + ½5.76\cdot(2.5\text{ cm/s})^2$ (50 erg = 50 erg)

we find that total kinetic energy is conserved ($1\text{ erg} = \text{g}\cdot\text{cm}^2/\text{s}^2$)

Since vectorially, momentum conservation implies,

$\mathbf{p_{1i}} = \mathbf{p_{1f}} + \mathbf{p_{2f}}$ the dot product of each side gives:

$$p_{1i}^2 = p_{1f}^2 + p_{2f}^2 + 2p_{1f}p_{2f}\cos\phi$$

Plug in numbers w/ p = mv, etc.

$$400.0 = 256 + 207.36 + 2(16)(14.4)\cos\phi$$

$$-63.36 = 460.8\cos\phi$$

Solving, $\cos\phi = -0.1375$, meaning angle must be greater than 90^o,

$$\phi = \cos^{-1}(-0.1375) = 98^o = \theta_1 + \theta_2$$

Suppose we now ask the trickier question, how are the departing masses oriented relative to east? It suffices to look at the X-component statement:

$m_1v_{1ix} = m_1v_{1fx} + m_2v_{2fx}$ conservation of momentum in X -direction

$$m_1(v_{1i}\cos(0)) = m_1v_{1f}\cos(\theta_1) + m_2v_{2f}\cos(\theta_2) = m_1v_{1f}\cos(\theta_1) + m_2v_{2f}\cos(98-\theta_1)$$

Let $m_1 = 4.00$ g then $m_2 = 5.76$ g $= 1.44m_1$ we have:

$$m_1v_{1i} = m_1v_{1f}\cos(\theta_1) + (1.44\,m_1)v_{2f}\cos(98-\theta_1)$$

so that we have, eliminating m's and inserting values of velocities

$$v_{1i} = v_{1f}\cos(\theta_1) + 1.44v_{2f}\cos(98-\theta_1)$$

$$(5\ cm/s) = (4\ cm/s)\cos(\theta_1) + 1.44(2.5\ cm/s)\cos(98-\theta_1)$$

$$1.25 = \cos(\theta_1) + 0.9\cos(98-\theta_1) \quad \text{[after dividing by 4 cm/s]}$$

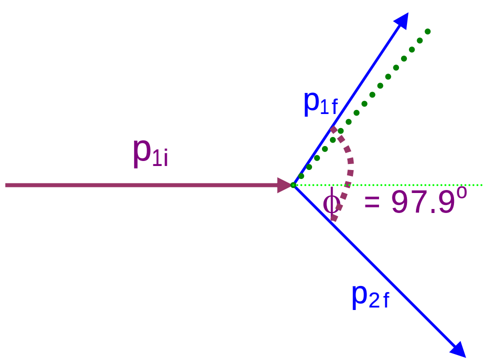
At this point, one could try to simplify the 2nd cosine function by:

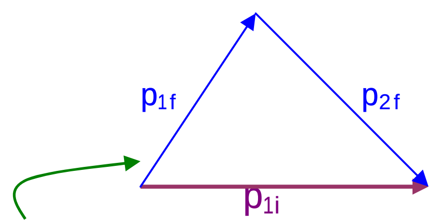
$$\cos(A \mp B) = \cos A\cos B \pm \sin A\sin B$$

and solve, or also employ the Y-component and solve a *non-linear* 2 equations/2 unknowns problem. This is b/c cosine and sine function depend non-linearly upon θ, as can be seen from their Taylor expansions. However, numerical solution or graphical construction may also be employed. One finds

$$\boldsymbol{\theta_1 = 40.6^o} \qquad \boldsymbol{\theta_2 = 57.3^o}$$

If we graphically construct the momenta vectors, one dashed line serves to guide the eye and convince one that the "exit" angle is greater than 90^o, and the other dashed line guides the eye to see the θ_1 and θ_2 relative to east.





This triangle show momentum conservation