DISCUSSION SECTION PROBLEM

For discussion yesterday, I had asked you to solve problem 53 on p. 189 in Chapter 7. Most people solved this using conservation of momentum, although the text asks you to solve the problem using Newton' s laws. In this scenario, a clay ball of 0.1 kg initially travels at 10 m/s toward a block (initially stationary) of 0.9 kg which rests on a frictionless surface. The clay hits and sticks to the block, exerting a constant force on the block during the 0.01 s it takes for the clay to come to rest with respect to the block. After this, they move together as a system.

You are asked to use Newton' s laws to find the speed of the block/clay system after impact.

To do this, we first realize that the force the clay exerts on the block will be equal in magnitude (and opposite in direction) to the force of the block on the clay. In other words, Newton' s third law tells us :

$$F_1 = -F_2 \text{ or } m_1 a_1 = -m_2 a_2 \tag{1}$$

where I will use subscript 1 to refer to the clay ball and subscript 2 to refer to the block.

We are also told that the block is initially at rest (with respect to the lab frame) and the clay is moving at 10 m/s. It is important to realize that after the collision both the clay and block are moving at the same speed, let' s call it v_f. We can write expressions for the accelerations of each mass:

$$a_1 = \frac{v_f - 10\,\mathrm{m/s}}{t} \qquad\qquad a_2 = \frac{v_f - 0}{t} = \frac{v_f}{t}$$

where t is the time in which the forces are exerted (and is 0.01 s for both objects). Recall that acceleration is the change in velocity divided by the time in which the velocity changes. Both objects have the same final velocity and the same time of contact. We can rewrite the acceleration of the clay as :

$$a_1 = \frac{v_f}{t} - \frac{10\,\mathrm{m/s}}{t} = a_2 - \frac{10\,\mathrm{m/s}}{t}$$

And use this result in equation (1) :

$$m_1\left(a_2 - \frac{10\,\mathrm{m/s}}{t}\right) = -m_2 a_2$$

Now we have an expression with only one unknown (the acceleration of the block); we know the masses and time of contact. Solving this for the acceleration of the block gives us :

$$a_2 = \frac{10\,m_1}{t\,(m_1 + m_2)} = \frac{10\,\mathrm{m/s} * 0.1\,\mathrm{kg}}{0.01\,\mathrm{s}\,(0.1\,\mathrm{kg} + 0.9\,\mathrm{kg})} = 100\,\mathrm{m}/\mathrm{s}^2$$

This tells us that the block accelerated from rest at this rate for 0.01 s; its velocity (and thus the velocity of the system) at the end of this 0.01 s interval is just

$$v_f = a\,t = 100\,\mathrm{m}/\mathrm{s}^2 * 0.01\,\mathrm{s} = 1\,\mathrm{m/s}$$