

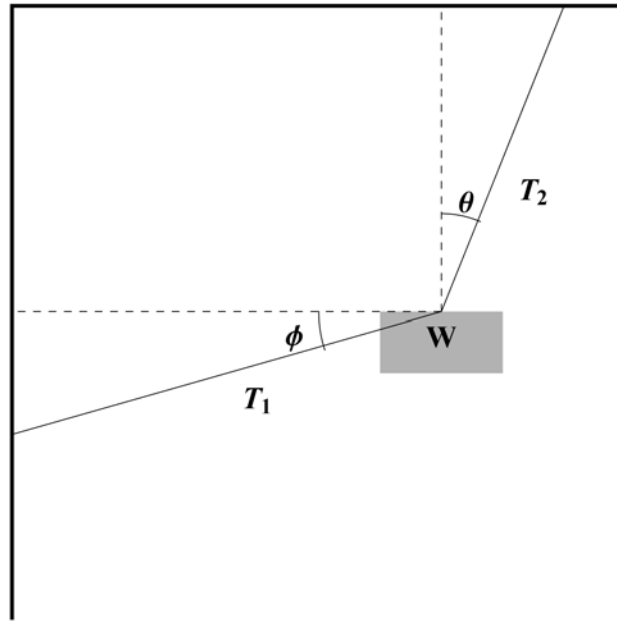
**PHYS 111 K**  
**SECOND HOUR EXAM**  
**2015**

This is a closed book closed note exam. Do all your writing in your blue book (s) and be sure to put your name on each blue book you use. You will not need nor are permitted to use calculators or any other electronic device. Please put all electronic devices out of sight now.

You may do questions in any order; it is a good idea to read over the test first and start with those questions in which you are most confident. A list of equations and results is at the end of the exam.

You must provide complete solutions for each question and show clearly the steps you have taken to reach your solution. Answers without work, even if they are correct, will receive no credit.

1. A weight of magnitude  $W$  is suspended in equilibrium by two massless cables as shown in the diagram below. The tensions in each cable are shown, as are the angles of the cables. Apply Newton's second law to this situation, and write the sum of forces in both the  $x$  and  $y$  directions. Determine expressions for  $T_1$  and  $T_2$  (of course, you can't solve for their numerical values). (20)



**Solution** : Since the system is in equilibrium, all the horizontal forces sum to zero, and all the vertical forces sum to zero. (Notice that the vertical component of  $T_1$  acts down.) This means that :

$$\Sigma F_x = T_2 \sin \theta - T_1 \cos \phi = 0 \Rightarrow T_2 = T_1 \frac{\cos \phi}{\sin \theta} \quad (1)$$

$$\Sigma F_y = T_2 \cos \theta - T_1 \sin \phi - W = 0 \Rightarrow T_2 \cos \theta = T_1 \sin \phi + W \quad (2)$$

Using substituting eq. (1) into eq. (2) yields :

$$T_1 \frac{\cos \phi}{\sin \theta} \cos \theta = T_1 \sin \phi + W$$

Collect terms and rearrange to find :

$$T_1 = \frac{W}{\cot \theta \cos \phi - \sin \phi}$$

Many students put the weight of the object into the horizontal force equation.

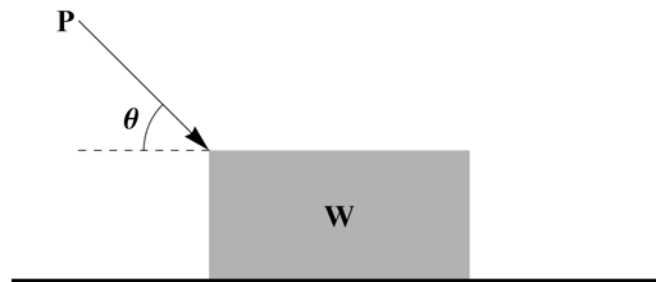
2. A force  $P$  acts on a crate of weight  $W$  as shown in the diagram below. The coefficient of kinetic

friction between the floor and the crate is  $\mu_k$  and the coefficient of static friction is  $\mu_s$ . The crate is stationary.

a) Draw a diagram of this situation identifying all forces acting on the crate (in both horizontal and vertical directions). Your diagram should show direction in which each force acts. (5)

b) Apply Newton's second law to this situation and derive an expression for the minimum force  $P$  which will get the crate into motion. (Write an expression for  $P$  in terms of  $W$ ,  $\mu$  and  $\theta$  (Make sure you use the correct coefficient of friction). (10)

c) What is the angle  $\theta$  for which motion is impossible? (i.e., there is no force sufficiently strong to get the crate into motion). (10)



**Solution :** Again we sum forces in the horizontal and vertical directions. Since the block has yet to move, we use the coefficient of static friction. Note : because there is a component of  $P$  that acts vertically, the normal force will not equal the weight of the crate. Many students made this error.

$$\Sigma F_x = P \cos \theta - f = 0 \quad (3)$$

$$\Sigma F_y = N - P \sin \theta - W = 0 \Rightarrow N = P \sin \theta + W \quad (4)$$

Now, since the force of friction is  $\mu_s N$ , equation (3) becomes :

$$P \cos \theta - f = P \cos \theta - \mu_s (P \sin \theta + W) = 0$$

or :

$$P (\cos \theta - \mu_s \sin \theta) = \mu_s W \Rightarrow P = \frac{\mu_s W}{\cos \theta - \mu_s \sin \theta}$$

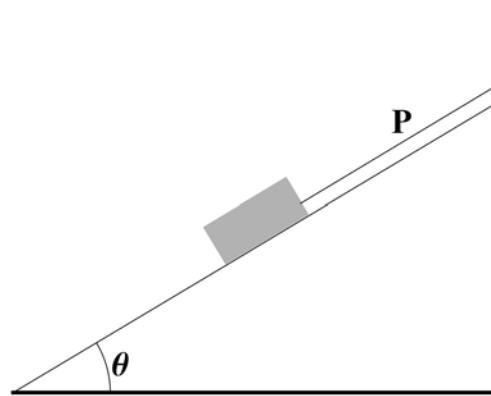
If the denominator goes to zero, the required force will go toward infinity. This situation occurs when :

$$\cos \theta - \mu_s \sin \theta = 0 \text{ or when } \mu_s = \frac{\cos \theta}{\sin \theta}$$

3. A block of mass  $m$  is pulled up an inclined plane of angle  $\theta$  at a constant speed by a force  $P$  which acts parallel to the plane. The coefficient of friction between the block and the plane is  $\mu$ .

a) Draw a diagram showing all forces acting on the block (5)

- b) Write Newton's second law for the forces parallel to the plane and also for the forces perpendicular to the plane. (5)
- c) Find an expression for the force needed to pull the block up the plane at constant speed in terms of  $m$ ,  $g$ ,  $\mu$  and  $\theta$ . (10)



**Solution** : Since the block is moving at constant speed up the incline, the sum of all forces up the incline must sum to zero. Our force equations become :

$$\Sigma F_{\parallel} = P - m g \sin \theta - \mu_k m g \cos \theta = 0$$

$$\Sigma F_{\perp} = N - m g \cos \theta = 0$$

Along the plane,  $P$  pulls up the plane, while a component of gravity acts down the plane. Friction acts down, since it opposes the block's motion up the plane. Therefore, these equations give us :

$$P = m g (\sin \theta + \mu_k \cos \theta)$$

4. Susan drives north (with respect to observer A who is standing by the side of the road) toward an intersection at 60 mi/hr. Trent drives east at 80 mi/hr (with respect to the same observer) toward the same intersection.

a) Write an expression for Trent's speed relative to Susan's reference frame using the subscript notation (as modeled in equation (3) in the list of equations at the end of this exam). (10)

b) What is Trent's speed relative to Susan's reference frame? (10)

(Provide the numerical answer; this is a simple calculation that you can do without the aid of a calculator. If you cannot, set up the calculation leaving it at the point you would resort to a calculator).

**Solution** : Using the subscript notation developed in the book and used in class, we can express Trent's velocity measured in Susan's reference as :

$$\mathbf{V}_{TS} = \mathbf{V}_{TG} + \mathbf{V}_{GS}$$

where  $\mathbf{V}_{TS}$  is Trent's velocity as measured by Susan,  $\mathbf{V}_{TG}$  is Trent's velocity with respect to the ground, and  $\mathbf{V}_{GS}$  is the ground's velocity with respect to Susan.

You could also reason that according to Susan, Trent would be traveling east at 80 mi/hr, and also traveling to the south at 60 mi/hr. Thus, Susan would observe Trent's vector velocity as:

$$\mathbf{V}_{TS} = 80 \hat{x} - 60 \hat{y}$$

(where I have set north and east as positive). Thus, the magnitude of this vector is :

$$\sqrt{(80 \text{ mi/hr})^2 + (-60 \text{ mi/hr})^2} = 100 \text{ mi/hr}$$

Most students ignored the vector nature of velocity and added the components, obtaining a speed of 20 mi/hr.

5. A projectile of mass  $m$  is launched from the ground with a launch velocity of  $v_o$  at an angle  $\theta$  with respect to the level ground. In this case, we will consider air drag acting on the projectile, and will use a simplified (albeit not very realistic) model for atmospheric drag. Let's assume that the drag force acting on the projectile can be expressed as :

$$\mathbf{D} = k \mathbf{v}$$

where  $k$  is a constant of proportionality. This means that the vector force of drag ( $\mathbf{D}$ ) is proportional to the instantaneous velocity vector ( $\mathbf{v}$ ) of the projectile. Write the equations of motion (i.e.,  $x(t)$  and  $y(t)$ ) for the projectile now taking into account the presence of air drag. I am not asking you to solve these equations in any way, merely to adapt the equations of motion to include this drag force. (Remember that drag and velocity are vector quantities). (15)

**Solution :** The significant issue here is that forces cause accelerations. The drag force acting on the projectile will cause it to accelerate, and we can express the components of that acceleration using :

$$D_x = -k v_x = m a_x \Rightarrow a_x = \frac{-k v_x}{m}$$

The drag force acting in the vertical direction will cause an additional acceleration (additional to gravity) of :

$$D_y = -k v_y = m a_y \Rightarrow a_y \text{ (due to drag)} = \frac{-k v_y}{m}$$

We use these expressions for acceleration in the equations of motion :

$$x(t) = x_0 + v_o \cos \theta t - \frac{1}{2} \left( \frac{k v_x}{m} \right) t^2$$

$$y(t) = y_0 + v_o \sin \theta t - \frac{1}{2} \left( g + \frac{k v_y}{m} \right) t^2$$

This is all you had to do on this problem.

**List of Equations and Results**

$$\Sigma \mathbf{F} = m \mathbf{a} \quad (5)$$

$$f = \mu N \quad (6)$$

$$v_{AB} = v_{AC} + v_{CB} \quad (7)$$

$$x(t) = x_0 + v_{0x} t + \frac{1}{2} a_x t^2 \quad (8)$$

$$\mathbf{W} = m \mathbf{g} \quad (9)$$

$$a^2 + b^2 = c^2 \quad (10)$$