PHYS 111 K
THIRD HOUR EXAM
SOLUTIONS

1. The planet has mass m, the star has mass M, the orbital distance between their centers is R. The force of gravity between the two masses is :

$$F_{Grav} = \frac{G\,m\,M}{R^2}$$

This force generates the centripetal force on the planet, so we can equate those forces to find:

$$\frac{G\,m\,M}{R^2} = \frac{m\,v^2}{R} \Rightarrow v^2 = \frac{G\,M}{R}$$

Since the planet revolves in a circular orbit at constant speed, we know that the time to complete one revolution (the period) is related to the circumference of the orbit by:

$$P = \frac{2\,\pi\,R}{v} \Rightarrow v = \frac{2\,\pi\,R}{P} \Rightarrow v^2 = \frac{4\,\pi^2\,R^2}{P^2}$$

Substitute this expression into the v^2 equation above:

$$\frac{4\,\pi^2\,R^2}{P^2} = \frac{G\,M}{R}$$

A little algebra yields:

$$M\,P^2 = \left(\frac{4\,\pi^2}{G}\right) R^3$$

from which it follows that $P \propto R^{3/2}$. This is the statement of Kepler's Third Law.

2. This problem is worked out in detail in the solutions for HW 7, problem no. 1.

3. This problem is worked out in detail in the text. The most common (and quite common error) students made was in evaluating the integral :

$$-\int_{x_i}^{x_f} k\,(\Delta x)\,dx = -\int_{x_i}^{x_f} k\,(x - x_o)\,dx$$

where k is the spring constant, x is the instantaneous displacement of the spring, x_o is the equilibrium distance, and x_i and x_f are the initial and final values of the displacement of the spring. Be careful to note that you are integrating with respect to x and not Δ x (many students simply wrote:

$$\int (\Delta x)\, dx = \frac{(\Delta x)^2}{2}$$

this is not correct. Remember that $\Delta x = x - x_o$, so the integral is:

$$\int (x - x_o)\, dx = \frac{x^2}{2} - x_o\, x \text{ (and we have to evaluate at appropriate constants)}$$

We could integrate this function this way, or make use of a simple substitution:

$$u = x - x_o$$

When making use of the " u substitution method", you must transform every element of the integral. Since $u = x - x_o$, $du = dx$. We must also convert the limits to the new variable:

$$u_f = x_f - x_o \quad \text{and } u_i = x_i - x_o$$

Now, since we have defined $\Delta x = x - x_o$, we have that $\Delta x_f = x_f - x_o$ and also $\Delta x_i = x_i - x_o$ so that our integral becomes:

$$-k \int_{\Delta x_i}^{\Delta x_f} u\, du = \left.\frac{-k\, u^2}{2}\right|_{\Delta x_i}^{\Delta x_f} = \frac{-k}{2}\left[(\Delta x_f)^2 - (\Delta x_i)^2\right]$$

4. The statement that the rain falls vertically means that the rain does not contribute to the horizontal momentum, so that there are no horizontal forces acting on the rain/car system. This means that momentum is conserved. The momentum before the rain falls is M V. The momentum of the car/rain system after a mass of rain m has accumulated is $(m + M)\, V_{\text{after}}$. Equating these (since momentum is conserved) yields:

$$V_{\text{after}} = \frac{M\, V}{m + M}$$

This problem was worked out in detail in exactly this form in discussion.

5. This problem was worked out in detail in the text (p. 208) and homework 8 problem no. 5.