PHYS 111
HOMEWORK #3
Solutions

Remember to write on only one side of the paper. I will begin assessing penalties for homeworks not meeting the format specifications outlined in the syllabus.

1. Refer to problem 6 in the text, p. 61. Do parts a) and b). Then write a short paragraph describing (in prose) the trajectory defined by the graph. What is the average velocity for the trip? Five pts each part, 20 points for the entire question.

Solutions : In each case, we will compute average velocity from :

$$v_{av} = \frac{x_2 - x_1}{t_2 - t_1}$$

For the trip from A to B, we have :

$$v_{av}\,(A \to B) = \frac{25\,m - 0\,m}{3\,s - 0\,s} = 8.3\,m/s$$

For the trip from B to C :

$$v_{av}\,(B \to C) = \frac{0\,m - 25\,m}{6\,s - 3\,s} = -8.3\,m/s$$

For the trip from A to C, we could recongize that this is just the sum of the trip from A → B and B → C ⇒ average velocity is zero, or we could apply the formula :

$$v_{av}\,(A \to C) = \frac{0\,m - 0\,m}{6\,s - 0\,s} = 0$$

Average velocity is the time rate of change of net displacement; average speed is the time rate of change of total (scalar) distance traveled. The average speed is the same as the average velocity for the segment A → B. The average speed for the segment B → C is 8.3 m/s (without the negative sign, since speed describes only the magnitude of motion). The average speed for the entire trip is 50 m/6 s or 8.3 m/s, since the total distance traveled is 50 m in the total time interval of 6 s.

The graph indicates that the object left the origin at t = 0 s, traveled to a distance x = 25 m at t = 3 s, then returned to the origin along the same path reaching the origin at t = 6 s.

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2. Your car' s brake system is such that you can come to rest from an initial velocity V in a distance X. If you start at initial velocity 2 V, how far would you travel before your brakes brought you to

rest? Make sure your answer shows your procedure completely.

Solution : This problem involves the use of proportional reasoning. The relevant equation for this case is :

$$v_f^2 = v_0^2 + 2\,a\,d$$

where v_f is the final velocity of the object, v_o is the initial object, a is the acceleration and d is the total distance traveled. If the object comes to rest, we know its final velocity is zero, so we can solve for stopping distance via :

$$0 = v_0^2 + 2\,a\,d \Rightarrow d = \frac{-v_0^2}{2\,a}$$

Since the object is slowing down, the acceleration is negative. We see from the equation above that stopping distance, d, varies as the square of the initial velocity. Therefore, if we double the initial velocity, the stopping distance will increase by a factor of $2^2 = 4$, so the stopping distance in this case is 4 X.

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3. Problem 22 in text, p. 62.

Solution : Since we are given a graph of velocity vs. time, we know the slope of the tangent line to the curve yields the value of the instantaneous acceleration. Since the curve in question is a piece-wise linear (straight line) function, we need only measure the slope along any part of the appropriate straight line and that will be the slope, hence the instantaneous acceleration, of that portion of the trip.

Thus, at t = 3 s, we can see that the velocity is constant, hence the instaneous acceleration is zero. At t = 7 s, we can compute the slope by using easily read values at t = 6 s and t = 8 s :

$$a_{inst} = \text{slope of line} = \frac{v(8\,s) - v(6\,s)}{8\,s - 6\,s} = \frac{35\,m/s - 25\,m/s}{2\,s} = 5\,m/s^2$$

At t = 11 s we can use values at t = 10 s and at t = 12 s :

$$a_{inst} = \text{slope of line} = \frac{v(12\,s) - v(10\,s)}{12\,s - 10\,s} = \frac{10\,m/s - 35\,m/s}{2\,s} = -12.5\,m/s^2$$

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4. Problem 40, text, p. 63.

Solution : For this problem, we need to divide the total trip into three segments. Along the way, we will need to calculate some intermediate data to complete the determination of total distance traveled. For each trip, we will use the appropriate forms of the equations :

$$x(t) = x_0 + v_0\,t + \frac{1}{2}\,a\,t^2 \qquad v(t) = v_0 + a\,t$$

For the first segment, we are told the train starts from test; thus we can set initial position and veloc-

ity equal to zero and find that :

$$x_1 = 0 + 0 + \frac{1}{2} \cdot 1.6 \frac{m}{s^2} \cdot (14\,s)^2 = 156.8\,m$$

This is the distance covered by the train in the first segment (acceleration phase) of the trip. For the second phase, the train travels a ta constant speed for 70 s; we need to calculate how fast the train was traveling at the beginning of this phase (i.e., at the end of the acceleration phase). We are told it took 14 s to reach this constant velocity, so we have :

$$v\,(14\,s) = 0 + 1.6 \frac{m}{s^2} \cdot 14\,s = 22.4\,m/s$$

Traveling at this speed for 70 s, the train covers a distance :

$$x_2 = 22.4 \frac{m}{s} \cdot 70\,s = 1568\,m$$

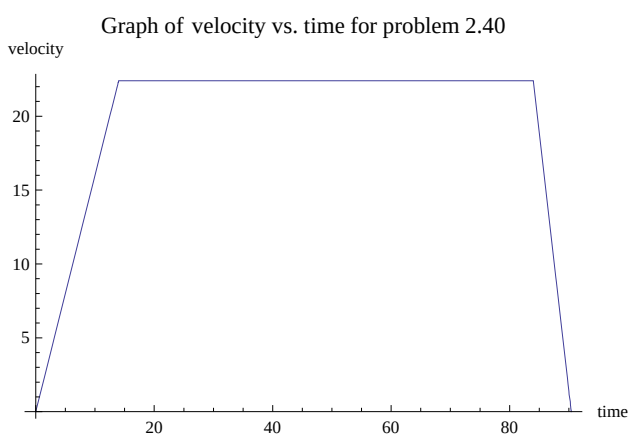
Finally, the train slows from 22.4 m/s to 0 m/s at the constant rate of 3.50 m/s/s. This means the final phase (negative acceleration phase) took a time equal to :

$$a_{avg} = \frac{v_2 - v_1}{t} \Rightarrow t = \frac{v_2 - v_1}{a_{av}} = \frac{0\,m/s - 22.4\,m/s}{-3.5\,m/s/s} = 6.4\,s$$

Knowing this time, we can compute the distance traveled in the last segment from :

$$x_3 = 22.4\,m/s \cdot 6.4\,s - \frac{1}{2} \cdot 3.5 \frac{m}{s^2} \cdot (6.4\,s)^2 = 71.7\,m$$

The total distance traveled is the sum of these three segments, or 1796 m. Below is a graph of the velocity vs. time for this situation. What would be the units of velocity · time? What is the area under the graph, and what are its units?



5. This is often called the ' Monk and the Monastery' problem in mathematical logic. A monk leaves his office in a city one day exactly at noon and walks at a constant speed to the monastery on the outskirts of town which is a distance D from the city. Being a beautiful day, he walks slowly arriving at his destination at 7 pm. The next day, he leaves the monastery at exactly noon, and

walks back to his office in the city (at a faster constant speed) along exactly the same path he took the day before. However, since the skies appear threatening, he walks rapidly, arriving at his office at 3 pm.

Will there be any point along the trip that he passes at the same time on the two days? Explain why or why not. (Drawing a graph of the two trips on the same set of axes might be very useful.) If there is such a point, determine how far from the city it is, and the time in which he reaches it on each day.

Solution : The simplest (but very clever) way to think about this situation is to imagine that there are two travelers starting out at noon on the same day. If one starts at the monastery and one from the city and they both begin their journeys at noon, then it is necessary that they meet somewhere along the path. We can figure out exactly where this intersection occurs by writing the equation of motion for each traveler. Since they are walking at constant speeds, their equations of motion are simply :

$$x(t) = x_0 + v\,t$$

(there are no acceleration terms since the motion is at constant velocity). Now, if we call the city the zero point of our x coordinate, the outbound traveler has an initial x value of 0, but the inbound (starting at the monastery traveler) has an initial x position of D. If the outbound traveler completes the trip in 7 hrs, his velocity was D/7; the inbound traveler has a velocity of - D/3 (the minus sign arises because the motion is in the opposite direction) . Therefore, we can write for the two travelers :

$$x_{out}(t) = 0 + \frac{D}{7}t \qquad x_{in}(t) = D - \frac{D}{3}t$$

The travelers meet when their x coordinates are the same :

$$x_{out}(t) = x_{in}(t) \Rightarrow \frac{D}{7}t = D - \frac{D}{3}t \Rightarrow \left(\frac{D}{7} + \frac{D}{3}\right)t = D$$

Cancelling out a common factor of D yields :

$$(1/7 + 1/3)\,t = 1 \Rightarrow \frac{10}{21}t = 1 \text{ or } t = 2.1 \text{ hours}$$

The travelers meet 2.1 hours after they depart (or at 2 : 06 pm). To find where this is along the path, substitute this time in either equation of motion :

$$x_{out}(t = 2.1) = \frac{D}{7}\cdot 2.1 = 0.3\,D$$

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6. Consider a river with parallel banks that lie exactly on the x axis. A boat that can travel at a speed v in still water travels in a river of current V. The current is directed exactly along the - x axis. The boat travels a distance D downstream (in the direction of the current), stops, and returns to its starting point. Show that the time to complete the round trip is :

$$\text{time} = \frac{2\,v\,D}{v^2 - V^2}$$

assume that no time elapses when the boat is making its turn. What is the time to complete a round trip in the case where v = V? Provide a physical explanation for this result. (In discussion we will spend time delineating the difference between a physical and mathematical explanation.) (10 pts for derivation, 5 pts for explanation)

Solution : The time to complete the round trip is the sum of the time to go a distance downstream and the time it takes to go a distance upstream. Traveling at a constant velocity, the time required to travel a distance t is simply t = D/v. For the downstream trip, the boat is moving in the direction of the current, so its velocity with respect to the shore is v + v; going upstream, the velocity with respect to the shore is v - V. Therefore, the total time of the roundtrip is :

$$\text{time for total trip} = \frac{D}{v+V} + \frac{D}{v-V}$$

Adding these algebraically gives us the result :

$$\text{total time} = \frac{D\,(v-V) + D\,(v+V)}{(v+V)\,(v-V)} = \frac{2\,v\,D}{v^2 - V^2}$$

When v = V, the denominator goes to zero and the time to make the roundtrip tends toward infinity. Why does this happen? If the current speed opposing the motion of the boat equals the speed of the boat in still water, the boat will have a net velocity of zero with respect to the shore, and will never move upstream.

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7. A rock is dropped from rest from a cliff of height H above a well. After a time T elapses (from the moment when the rock is dropped), the sound of the rock splashing into the water is heard by an observer at the top of the cliff. If the speed of sound is c (and is a constant), derive an expression for the height of the cliff, H, in terms of T, c, and g (the acceleration due to gravity). We ignore the effects of air friction for this problem. If the speed of sound is 343 m/s and the splash is heard 9.37 s after the rock was dropped, determine the height of the cliff. (15 pts for derivation, 5 pts for calculation). Use g = 9.8 m/s/s

Solution : As in the problem above, we recognize that the time it takes for the sound of the rock to reach the observer (listener?) is the sum of the time it takes for the rock to fall to the water and the time it takes the sound wave to travel back to the top of the cliff. Thus we write :

$$T = t_1 + t_2$$

where T is the total time, t_1 is the time it takes for the rock to fall to the water and t_2 is the time it takes the sound wave to travel the distance H to the listener. We can determine these times by appealing to the equation of motion in one dimension:

$$y(t) = y_0 + v_0\,t + \frac{1}{2}\,a\,t^2$$

For the first leg of the trip (the rock falling), let' s set the initial y value as H, the initial velocity is zero (since the rock falls from rest) and the acceleration is - g (if we set the top of the cliff to y = H

and the surface of the water to y = 0, we are establishing a coordinate system in which up is the positive direction, so that the vector g acts in the negative direction). Thus, we can find the time it takes the rock to reach the surface of the water (y = 0) from :

$$0 = H - \frac{1}{2} g t_1^2 \Rightarrow t_1 = \sqrt{\frac{2H}{g}}$$

Now, the sound wave will travel at a constant speed c, so it will take a time :

$$t_2 = \frac{H}{c}$$

for the sound wave to return to the top of the cliff. We have then :

$$T = t_1 + t_2 = \sqrt{\frac{2H}{g}} + \frac{H}{c}$$

We are asked to find an expression for H, and we can do this a couple of ways. One would be to recognize that the equation directly above is a quadratic equation in $\sqrt{H}$, and solve that quadratic fo $\sqrt{H}$. We can also write:

$$T = \sqrt{\frac{2H}{g}} + \frac{H}{c} \Rightarrow T - \frac{H}{c} = \sqrt{\frac{2H}{g}}$$

Squaring both sides gives :

$$\left(T - \frac{H}{c}\right)^2 = \frac{2H}{g} \Rightarrow T^2 - \frac{2HT}{c} + \frac{H^2}{c^2} = \frac{2H}{g}$$

rewriting in the standard form :

$$\frac{H^2}{c^2} - 2H\left(\frac{T}{c} + \frac{1}{g}\right) + T^2 = 0$$

where we recognize to be a quadratic in H. Solving using the quadratic equation :

$$H = \frac{2\left(\frac{T}{c} + \frac{1}{g}\right) \pm \sqrt{4\left(\frac{T}{c} + \frac{1}{g}\right)^2 - \frac{4T^2}{c^2}}}{\left(2/c^2\right)}$$

I will give full credit for getting this far, and not require that you do any further simplificaton of this solution. If you expand the discriminant (the stuff inside the square root), you get :

$$4\left(\frac{T}{c} + \frac{1}{g}\right)^2 - \frac{4T^2}{c^2} = 4\left[\frac{T^2}{c^2} + \frac{2T}{cg} + \frac{1}{g^2}\right] - \frac{4T^2}{c^2} = 4\left(\frac{2T}{cg} + \frac{1}{g^2}\right)$$

Pulling the factor of 4 outside of the square root yields :

$$H = \frac{2\left(\frac{T}{c}+\frac{1}{g}\right) \pm \sqrt{4\left(\frac{T}{c}+\frac{1}{g}\right)^2 - \frac{4T^2}{c^2}}}{\left(2/c^2\right)} = \frac{2\left(\frac{T}{c}+\frac{1}{g}\right) \pm 2\sqrt{\frac{2T}{cg}+\frac{1}{g^2}}}{\left(2/c^2\right)} =$$

$$c^2\left[\left(\frac{T}{c}+\frac{1}{g}\right) \pm \sqrt{\frac{2T}{cg}+\frac{1}{g^2}}\right]$$

Now, to substitute the given values and determine the height of the cliff :

$$H = (343\,m/s)^2\left[\frac{9.37\,s}{343\,m/s}+\frac{1}{9.8\,m/s^2} \pm \sqrt{\frac{2\,(9.37\,s)}{343\,m/s\cdot 9.8\,m/s^2}+\left(\frac{1}{9.8\,m/s^2}\right)^2}\right]$$

$$H = (343\,m/s)^2\left[0.0273\,\frac{s^2}{m}+0.1020\,\frac{s^2}{m} \pm \sqrt{0.00556\,\frac{s^4}{m^2}+0.0104\,\frac{s^4}{m^2}}\right]$$

$$H = 30\,095\,m \text{ or } 343\,m$$

How do we know which solution is realistic? If we dropped an object from 30, 000 m, it would take (neglecting friction)

$$t = \sqrt{2\cdot 30\,000\,m/9.8\,m/s^2} = 78\,s \text{ to fall,}$$

so if the total time was only a bit more than 9 seconds, we know that the smaller solution of approximately 343 m is correct. Your exact numerical solution might differ from this by a bit, since we have to take the difference of two small numbers to produce a final answer. Your result will depend sensitively on how many significant figures you retained during your intermediate calculations.