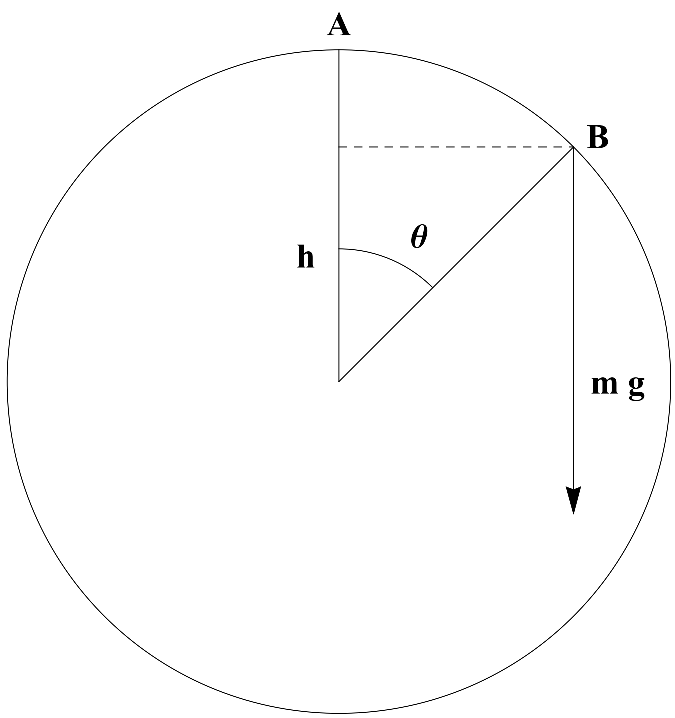
PHYS 111 K
HOMEWORK #10

Solutions

1. This problem combines concepts of energy conservation and circular motion. Consider a particle that starts from rest at the top of a frictionless vertical circle of radius R (at position A) and slides toward position B.

Solution : Let' s start by considering the diagram below.



a) We will employ conservation of energy principles to determine the speed at B. We will use the center of the circle as our reference level, this means that we will make all potential energy measurements with respect to this level. At A, the object is at rest so has no kinetic energy, and has potential energy equal to m g R (since it is a distance R above the center of the circle). At point B, the particle has lost some amount of potential energy, which (since there is no friction) will be converted into kinetic energy. Thus, we write the conservation of mechanical energy :

$$U_A + K_A = U_B + K_B$$

When the particle is at B, it is a height h above the reference level, we can use the geometry of the diagram to easily deduce that h = R cos θ, so we can rewrite the conservation of energy as :

$$m\,g\,R + 0 = m\,g\,R\cos\theta + \frac{1}{2} m\,v^2$$

Combining terms and dividing out a common factor of m :

$$v^2 = 2\,g\,R\,(1 - \cos\theta)$$

b) Consider the particle at B : The forces acting along the radial are the normal force out (the normal force always acts perpendicular to the point of contact), the component of weight along the radial, and these two forces combine to produce a centripetal force acting toward the center of the circle. Setting the direction of the outward normal as positive, we have then :

$$N - m\,g\cos\theta = \frac{-m\,v^2}{r}$$

c) Now, use the expression for v from part a) in the force equation and we get :

$$N = m\,g\cos\theta - m\,\frac{(2\,g\,R\,(1 - \cos\theta))}{R} \Rightarrow N = m\,g\,(3\cos\theta - 2)$$

The ball will slide off the circle when the normal force goes to zero, or when

$$\cos\theta = 2/3$$

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2. Problem 53, p. 275.

Solution : First, we need to determine the minimum velocity a particle can have and still execute a complete vertical circle. Then, once we know this speed, we can use energy methods to determine the height from which it must be dropped.

At the top of a vertical circle, the normal force acts down (the particle moves along the inside track), weight acts down, and these produce a centripetal force that acts toward the center of the circle. Newton' s second law gives :

$$-N - mg = \frac{-m\,v^2}{r} \Rightarrow N = m\left(g - \frac{v^2}{R}\right)$$

The particle will maintain contact with the circle as long as N > 0, so the minimum speed required is :

$$v^2 = R\,g$$

Now, we use conservation of energy. At the top of the the slide, the particle is a height h above our reference level (let' s use the surface) and is at rest so has no kinetic energy. At the top of the circle, it is a height 2 R above the reference level, and is moving at the speed we just determined. Conservation of energy tells us :

$$m\,g\,h + 0 = m\,g\,(2\,R) + \frac{1}{2}\,m\,v^2$$

But we know that v^2 = R g, so that:

$$m\,g\,h = 2\,m\,g\,R + \frac{1}{2}\,m\,g\,R \Rightarrow h = \frac{5}{2}\,R$$

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3. Problem 49, p. 274.

Solutions :

a) This is a head on, perfectly elastic collision with one object at rest. We can compute the velocities of the two objects using the equations derived in the text (10.42 on p. 266)

$$v_{\text{ball, final}} = \frac{m_{\text{ball}} - m_{\text{block}}}{m_{\text{ball}} + m_{\text{block}}}\,v_{\text{ball, initial}} = \frac{0.02\text{ kg} - 0.1\text{ kg}}{0.02\text{ kg} + 0.1\text{ kg}} \cdot 5\text{ m/s} = -3.33\text{ m/s}$$

the minus sign indicating that the ball’s final velocity is in the negative direction (having called its initial direction positive).

b) We use conservation of energy methods to find the maximum compression of the block. Immediately after the collision, the block has kinetic energy but no elastic potential energy (since the spring is at equilibrium length). At maximum compression, all the original KE has converted to EPE, so we can write:

$$\frac{1}{2}\,m\,v^2 = \frac{1}{2}\,k\,(\Delta x)^2$$

First we must compute the initial velocity of the block. We do so using the equation found on p. 266 :

$$m_{\text{block}} = \frac{2\,m_{\text{ball}}}{m_{\text{ball}} + m_{\text{block}}}\,v_{\text{ball, initial}} = \frac{2\,(0.02\text{ kg})}{0.02\text{ kg} + 0.1\text{ kg}} \cdot 5\text{ m/s} = 1.67\text{ m/s}$$

Then, using the data supplied in the problem :

$$\Delta x = \sqrt{m\,v^2 / k} = \sqrt{\frac{0.1\text{ kg} \cdot (1.67\text{ m/s})^2}{20\text{ N/m}}} = 0.12\text{ m}$$

c) In the case of a perfectly inelastic collision, the ball sticks to the block creating a single mass, and we use momentum conservation to determine the speed of the combined mass immediately after collision. Using m for the balls' s and M for the block' s mass :

$$m\,v = (M + m)\,V_{\text{after}} \Rightarrow V_{\text{after}} = \frac{m\,v}{m + M} = \frac{0.02\text{ kg}\,(5\text{ m/s})}{0.12\text{ kg}} = 0.83\text{ m/s}$$

This is the speed of the combined mass immediately after collision. We again use energy methods to relate the total energy immediately after collision to the total energy at maximum compression. Since there is no EPE when the spring is uncompressed at the beginning, and no kinetic energy when the spring is fully compressed, we have :

$$\frac{1}{2}(m+M)V^2 = \frac{1}{2}k(\Delta x)^2 \Rightarrow \Delta x = \sqrt{(m+M)V^2/k} = \sqrt{(0.12\text{ kg})(0.83\text{ m/s})^2/20\text{ N/m}}$$

$\Delta x = 0.064$ m

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4. Problem 71, p. 276

Solution : There are some nuances to this problem we have to consider. First, gravitational potential energy is determined by the vertical distance above a reference level. We will need to remember this since our problem here is on an incline. Second, we need to recognize that the mass will not stop once it hits the spring but will compress the spring by some amount, and will lose some gravitational potential energy as it compresses the spring. In this problem then, we need to consider kinetic energy, gravitational potential energy and also elastic potential energy. If we call A the starting point of the mass and B its position when the spring is fully compressed, conservation of energy gives us :

$$K_A + U_{GA} + U_{EA} = K_B + U_{GB} + U_{EB}$$

where K represents kinetic energy, U_G gravitational potential energy (GPE) and U_E elastic potential energy. At the top (position A), there is neither kinetic nor elastic potential energy (EPE). To determine an expression for the GPE at A, we need to choose a reference level. We have a couple of choices here, but let’s choose the reference level to be the position where the spring is fully compressed. The spring will compress a distance Δx (we don’t know this distance but will figure it out), so the mass is initially 4+Δx from this point as measured along the incline of the plane. However, GPE is determined by the vertical distance between initial and final points, so the GPE at A is m g (4+Δx) sin θ where θ is the angle of the incline. Since we are choosing B to be our reference level, there is neither GPE nor KE there, and EPE given by 1/2 k $(\Delta x)^2$. Thus, our energy balance gives us:

$$m g (4+\Delta x)\sin\theta = \frac{1}{2}k(\Delta x)^2$$

This equation is a quadratic in Δx

$$\frac{1}{2}k(\Delta x)^2 - m g \sin\theta\,\Delta x - 4 m g \sin\theta = 0$$

This yields :

$$\Delta x = \frac{m g \sin\theta \pm \sqrt{(m g \sin\theta)^2 + 8 m g k \sin\theta}}{k}$$

Substituting numbers :

$$\Delta x = \left[(10\,\text{kg})\left(9.8\,\text{m}/\text{s}^2\right)\sin 30 \pm \sqrt{\left(\left(10\cdot 9.8\sin 30\,\text{kg m}/\text{s}^2\right)^2 + 8\,(10\,\text{kg})\left(9.8\,\text{m}/\text{s}^2\right)250\,\text{N}/\text{m}\right)}\right] \Big/ 250\,\text{N}/\text{m}$$

And this yields :

$$\Delta x = 1.46\,\text{m}$$

In the second part, we are asked to find how far the spring is compressed when the block has maximum speed. There are several approaches to this question, but I think this is the easiest solution. Let's write Newton's second law for the block once it is in contact with the spring. The forces acting on the block are the component of gravity down the plane (mg sin θ) and the spring force acting up the plane. Let's call down the plane positive, so we can write Newton's second law as:

$$\Sigma F = m\frac{dv}{dt} = m g \sin\theta - k x \tag{1}$$

Here, notice that we use the differential form of Newton' s law. So, how do we go from this formulation to figuring out where the block has greatest speed? You should have just learned in calculus class that an extremum occurs when the derivative is zero. If we set dv/dt = 0, we find that an extremum occurs when :

$$m g \sin\theta - k x = 0 \Rightarrow x = m g \sin\theta / k \tag{2}$$

or when :

$$x = 10\,\text{kg}\cdot 9.8\,\text{m}\big/\text{s}^2 \sin 30 / 250\,\text{N}/\text{m} = 0.2\,\text{m} \tag{3}$$

The block achieves maximum speed when the spring is compressed by 20 cm. (How can you be sure this is the maximum velocity (and not the minimum velocity)? What test do you apply and how do you apply it in this case?)

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5. Problem 72, p. 276

Solution : This is a problem that combines conservation of momentum with conservation of energy. We are asked to find the speed of the cannonball knowing that the recoil caused the cannon to compress a stiff spring by 0.5 m. Applying conservation of energy to the cannon/spring system :

$$K_i + U_i = K_f + U_f$$

The kinetic energy at the moment of firing is 1/2 MV^2 where M is the mass of the cannon and V is its recoil velocity. The initial potential energy is zero since the spring is uncompressed. In the final state, the cannon comes to rest so K_f is 0, and $U_f = 1/2\ k\ (\Delta x)^2$. Conservation of energy allows us to write:

$$\frac{1}{2} M V^2 = \frac{1}{2} k (\Delta x)^2 \Rightarrow V = (\Delta x)\sqrt{\frac{k}{M}}$$

We can relate V to the launch speed of the cannon ball by using conservation of momentum. Before the ball was fired, the cannon/cannonball system had zero momentum. Therefore, we know that the speeds of the cannon and cannonball immediately after firing must satisfy:

$$0 = m\mathbf{v} + M\mathbf{V}$$

where m is the mass of the cannonball and v is its launch velocity. Conservation of momentum implies that :

$$V = \frac{-m}{M} v$$

Use this relationship for v in the energy equation (and we can neglect the minus sign since we are solving for speed rather than velocity):

$$\frac{m}{M} v = (\Delta x)\sqrt{\frac{k}{M}} \Rightarrow v = (\Delta x)\frac{M}{m}\sqrt{\frac{k}{M}} = 0.5\,\text{m}\left(\frac{200\,\text{kg}}{10\,\text{kg}}\right)\sqrt{\frac{20\,000\,\text{N/m}}{200\,\text{kg}}} = 100\,\text{m/s}$$

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6. Problem 2, (bottom of p. 271)

Solution : Gravitational potential energy is :

$$U_{Grav} = m g h$$

where m is the mass of the person, g is the acceleration due to gravity and h is the height differential with respect to a reference level. In this case, we choose our reference level to be the base of Death Valley, so the height differential is 4420m-(-85m) = 4505m. The change in U is then:

$$\Delta U = 65\,\text{kg} \cdot 9.8\,\text{m}/\text{s}^2 \cdot 4505\,\text{m} = 2.9 \times 10^6\,\text{J}$$

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7. Problem 40, p. 273

Solution : In the first case, energy conservation tells us :

$$\frac{1}{2} k (\Delta x)^2 = \frac{1}{2} m v_0^2 \Rightarrow v = (\Delta x)\sqrt{\frac{k}{m}}$$

In the second case, the total spring constant is 2k (since there are two springs of equal stiffness), so energy conservation yields:

$$\frac{1}{2}(2\,k)(\Delta x)^2 = \frac{1}{2} m v^2 = (\Delta x)\sqrt{\frac{2\,k}{m}} = \sqrt{2}\, v_0$$

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8. Problem 24, p. 272

Solution : We apply conservation of energy to this situation. Just as the plane lands it has KE but no EPE. When the plane comes to rest, it has converted all its initial KE to EPE, thus we have :

$$K_i = U_f \Rightarrow \frac{1}{2} m v^2 = \frac{1}{2} k (\Delta x)^2 \Rightarrow v = (\Delta x) \sqrt{\frac{k}{m}} = 30\,\text{m} \sqrt{\frac{60\,000\,\text{N/m}}{15\,000\,\text{kg}}}$$

$$= 30\,\text{m} \sqrt{4/\text{s}^2} = 60\,\text{m/s}$$