PHYS 111 K
HOMEWORK #11-- SOLUTIONS

1. Note first that the motion of the crate is to the right in the diagram. For each force, we will use $W = \mathbf{F} \cdot \mathbf{r} = F\, r \cos\theta$ where θ is the angle between the force and direction of motion. Applying this we have :

$$W_1 = T_1\, r \cos 20 = 600\,\text{N} \cdot 3\,\text{m} \cos 20 = 1691\,\text{J}$$

$$W_2 = T_2\, r \cos 30 = 410\,\text{N} \cdot 3\,\text{m} \cos 30 = 1065\,\text{J}$$

$$W_3 = T_3\, r \cos 180 = -660\,\text{N} \cdot 3\,\text{m} = -1980\,\text{J}$$

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2. Since the distance we are moving is a considerable fraction of the radius of the Earth, the gravitational force exerted on the mass will vary during the trip, and we cannot set g equal to a constant. Thus, we cannot use $W = m\, g\, h$ for this problem. We use instead the integral form of work :

$$W = \int_{r_{initial}}^{r_{final}} \mathbf{F} \cdot \mathbf{dr} = \int_{R}^{2R} \frac{G\,M\,m}{r^2}\, dr = \int_{R}^{2R} \frac{G\,M\,m}{r^2}\, dr$$

$$W = \frac{-G\,M\,m}{r}\Big|_{R}^{2R} = -G\,M\,m\left(\frac{1}{2R} - \frac{1}{R}\right) = \frac{G\,M\,m}{2R}$$

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3. For this problem, we need to remember that force and potential are related by :

$$F = -\frac{dU}{dx}$$

Since the graph provided consists of a series of straight line segments, we can find the value of dU/dx by computing the slope of each straight line. The slope of the line at x = 5cm is -10J/10cm =-10J/0.1m
or the force has x component of +100N.

When the particle is at x = 15 cm, dU/dx =0 so the force is zero. At x= 35 cm, +10J/0.2m = 50N, so the x component of the force at x = 35 cm is - 50N.

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4. We take the derivative of the potential :

$$F_y = -\frac{dU_y}{dy} = -12\,y^2\ \text{J}/\text{m}$$

The force at each distance is:

$$F(0\ m) = -12\ (0)^2\ N = 0\ N$$

$$F(1\ m) = -12\ N$$

$$F(2\ m) = -48\ N$$

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5. Let's start with the frictionless case. Before motion ensues, the system has no kinetic energy and the falling block has potential energy of m g h. Assuming the mass on the table remains on the table, it experiences no change in potential energy, so we do not need to know its value of initial potential energy since we know it will equal the final value of potential energy. Energy conservation in the frictionless case yields :

$$m\,g\,h = \frac{1}{2}\,(m + M)\,v^2$$

where v is the speed of the system at the instant the block hits. Note that both blocks will be moving at the speed v, so both have kinetic energy. Simple algebra yields:

$$v = \sqrt{2\,m\,g\,h / (M + m)}$$

In the case with friction, the initial potential energy must equal both the final kinetic energy and the work lost to friction. The work done against friction will be equal to:
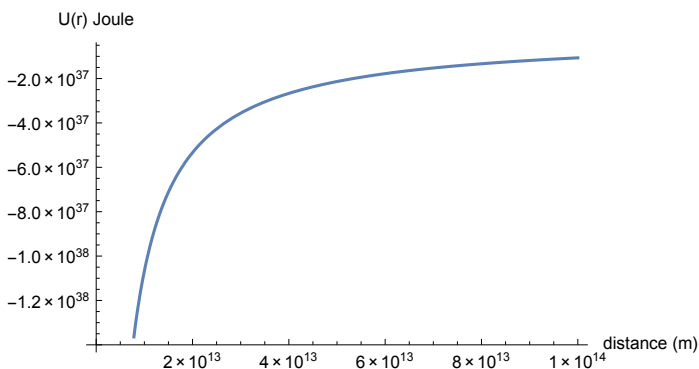
$$W_f = -\mu_k\,M\,g\,h$$

So, this amount of work reduces the amount of initial potential that can transform into kinetic and we have :

$$m\,g\,h - \mu_k\,M\,g\,h = \frac{1}{2}\,(M + m)\,v^2$$

or :

$$v = \sqrt{2\,g\,h\,(m - \mu_k\,M) / (m + M)}$$

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6. a) The graph below is produced using the data for m and r given in part c) :

The important points here are that the potential at infinity is zero (see part b), and as the objects near each other, their potential becomes more negative. This gives rise to the concept of a "potential well" ; think of an object falling deeper into a gravitational field as it moves deeper into the poten-

tial well (so that it requires more energy to lift it out of the well).

c) Since the objects start at rest, the initial kinetic energy is zero, and the change in potential must equal the sum of final kinetic energies. Conservation of energy gives us :

$$K_i + U_i \; = \; K_f + U_f$$

since the initial kinetic energy is zero, this becomes equivalent to:

$$K_f \; = \; U_i - U_f \; \text{or} : \quad \frac{1}{2} \, m_1 \, v_1^2 + \frac{1}{2} \, m_2 \, v_2^2 \; = \; - \, G \, m_1 \, m_2 \left(\frac{1}{r_i} - \frac{1}{r_f} \right)$$

Notice that we cannot assume the two stars will have the same speed at the end. We know all the values on the right of the equation above, but need a second equation to solve for the individual speeds. We find this by using the conservation of momentum. If we choose our system to be the two stars, the gravitational force between them is inside the system, and there are thus no external forces acting on the system. Thus, conservation of momentum yields:

$$0 \; = \; m_1 \, v_1 + m_2 \, v_2 \; \text{ or } v_2 \; = \; \frac{- \, m_1}{m_2} \, v_1$$

Substitute this into the equation above:

$$\frac{1}{2} \, m_1 \, v_1^2 + \frac{1}{2} \, m_2 \left(\frac{-m_1}{m_2} \, v_1 \right)^2 \; = \; - \, G \, m_1 \, m_2 \left(\frac{1}{r_i} - \frac{1}{r_f} \right)$$

We can solve for v_1:

$$v_1 \; = \; \sqrt{ -2 \, G \, m_2 \left(\frac{1}{r_i} - \frac{1}{r_f} \right) \Big/ \left(1 + \frac{m_1}{m_2} \right) }$$

Where r_i is the initial distance between the centers of the stars, r_f is the final distance between their centers (the sum of the stellar radii), m_1 and m_2 are the masses of the stars, and G is the Newtonian gravitational constant. Substituting those values I get v_1=689 km/s and v_2 = 172km/s

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7. We have essentially done this problem in #6. A rocket at the surface of the Earth experiences potential energy of magnitude G m M/R where G is the Newtonian Gravitational constant, m is the mass of the rocket, M is the mass of the Earth, and R is the radius of the Earth. If the rocket' s kinetic energy just exceeds this, the rocket can escape the Earth' s gravitational field. This condition is :

$$\frac{1}{2} \, m \, v^2 \; = \; \frac{G \, m \, M}{R} \Rightarrow v \; = \; \sqrt{ 2 \, \frac{G \, M}{R} } \; = \; \sqrt{ 2 \cdot 6.67 \times 10^{-11} \, \text{N} \, \text{m}^2 \, \text{kg}^{-2} \cdot 6 \, 10^{24} \, \text{kg} / 6.4 \, 10^8 \, \text{m} }$$

$$= \; \textbf{11.1 km / s}$$

In practice, it is not feasible to launch a rocket with escape velocity from the surface of the Earth. A rocket traveling through the atmosphere at this speed would experience so much drag that it would likely burn. Spacecraft are launched into low Earth orbit, and from there accelerated to escape

velocity.