PHYS 111 K
HOMEWORK #12-- SOLUTIONS

1. We always convert to SI (MKS) units, so we need to express rpm as rad/s. There are 2 π rad/rev and 60 s/min, so :

$$2000\ \mathrm{rev/min} = 2000\ \mathrm{rev/min} \cdot 2\pi\ \mathrm{rad/rev} \cdot 1\ \mathrm{min}/60\ \mathrm{s} = 209\ \mathrm{rad/s}$$

a) $\alpha = \dfrac{\Delta\omega}{\Delta t} = \dfrac{(209\ \mathrm{rad/s} - 0\ \mathrm{rad/s})}{0.5\ \mathrm{s}} = 418\ \mathrm{rad}/\mathrm{s}^2$

b) $\theta = \theta_o + \omega_o t + \dfrac{1}{2}\alpha t^2$

If the drill starts from rest, (and we assume our starting angle is zero), the angular displacement θ is:

$$\theta = \frac{1}{2}\alpha t^2 = \frac{1}{2}\left(418\ \mathrm{rad}/\mathrm{s}^2\right)(0.5\ \mathrm{s})^2 = 52.2\ \mathrm{rad}$$

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2. The equations for the center of mass are :

$$x_{cm} = \frac{x_1 m_1 + x_2 m_2 + x_3 m_3}{m_1 + m_2 + m_3} \qquad y_{cm} = \frac{y_1 m_1 + y_2 m_2 + y_3 m_3}{m_1 + m_2 + m_2}$$

where x, y refer to the positions of each particle and the m's are the masses of the particles.The 100 g mass is at (0,0); the 200g mass is at (0,10) and the 300g mass is at (10,0). Our center of mass equations tell us that:

$$x_{cm} = \frac{100\ \mathrm{g} \cdot 0 + 200\ \mathrm{g} \cdot 0 + 300\ \mathrm{g} \cdot 10\ \mathrm{cm}}{100\ \mathrm{g} + 200\ \mathrm{g} + 300\ \mathrm{g}} = 5\ \mathrm{cm}$$

$$y_{cm} = \frac{100\ \mathrm{g} \cdot 0 + 200\ \mathrm{g} \cdot 10\ \mathrm{cm} + 300\ \mathrm{g} \cdot 0}{600\ \mathrm{g}} = 3.33\ \mathrm{cm}$$

The coordinates of the center of mass are then (5, 3.33)cm

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3. The rotational kinetic energy of an object is

$$KE = \frac{1}{2} I \omega^2$$

where I is the moment of inertia and ω is the angular velocity. The moment of inertia of a sphere is :

$$I_{sphere} = \frac{2}{5} M R^2$$

so the rotational KE of the Earth is:

$$\text{KE} = \frac{1}{2}\,\text{I}\,\omega^2 = \frac{1}{2}\left(\frac{2}{5}\,\text{M}\,\text{R}^2\right)\omega^2 = \frac{1}{5}\,\text{M}\,\text{R}^2\,\omega^2$$

where M is the mass of the Earth, R is its radius, and ω is the angular velocity. We can find the angular velocity of the Earth knowing it completes $2\,\pi$ rad in one day, or:

$$\omega = \frac{2\,\pi}{1\text{ day}} = \frac{2\,\pi}{86\,400\text{ s}} = 7.3\times10^{-5}\text{ rad/s}$$

Substitute the appropriate values to find KE for the Earth.

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4. The moment of inertia of a disk for a rotation about its center is

$$\text{I}_{\text{disk}} = \frac{1}{2}\,\text{M}\,\text{R}^2 = \frac{1}{2}\,(0.021\text{ kg})\,(0.06\text{ m})^2 = 3.78\times10^{-5}\text{ kg m}^2$$

(remember to use the radius and to convert all values to SI units).

If the axis is perpendicular to the disk and goes through a point on the edge, we use the parallel axis theorem (not done in class, found on p. 321 of the text):

$$\text{I}_{\text{edge}} = \text{I}_{\text{disk}} + \text{M}\,\text{R}^2 = \frac{3}{2}\,\text{M}\,\text{R}^2 = 3\text{ times the value found just above.}$$

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5. We will find the net torque by summing the individual torques. The magnitude of each torque is F r sin θ where F is the magnitude of the force, r is the distance between the axis and the point where the force acts, and θ is the angle between F and r. We will also have to remember that torque is a vector and determine the direction (either clockwise (cw) or counterclockwise (ccw) of the torque.

1) The 30 N at the top of the disk causes a cw torque; since the force is perpendicular to the distance, the magnitude of this torque is 30 N · 0.1 m = 3 Nm

2) The other 30 N forces causes a ccw torque of magnitude 30 N· 0.05 m sin 45 = 1.06 Nm

3) The 20 N force in the lower right quadrant appears to act along a radius line of the disk. This means that the force generates no torque (since sin θ = 0).

4) The other 20 N force generates a ccw torque of magnitude 20 N · 0.05 m = 1.5 Nm

Thus, the total torque is (3 - 1.06 - 1.5) Nm = 0.44 Nm cw.

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6. We are interested in using the equation $\tau = \text{I}\,\alpha$ to find the torque necessary to cause the angular acceleration of the rod. First, we need to compute the moment of inertia, and before we do that, we need to find the center of mass of the object in order to compute the value of I. To make our life easy, we can imagine the rod lies along the x axis with one end (say the 1 kg ball) at x = 0. Then the 2 kg ball lies at x = 1 m, and the center of mass of this system is :

$$\text{x}_{\text{cm}} = \frac{1\text{ kg}\cdot0 + 2\text{ kg}\cdot1\text{ m}}{3\text{ kg}} = \frac{2}{3}\text{ m}$$

The center of mass, not suprisingly, is 2/3 of the way along the rod. Now, we compute the moment of inertia around this axis:

$$I = m_1 r_1^2 + m_2 r_2^2 = 1\,\text{kg}\,(2/3\,\text{m})^2 + 2\,\text{kg}\,(1/3\,\text{m})^2 = \frac{2}{3}\,\text{kg}\,\text{m}^2$$

In the calculation above, remember that the 1 kg mass is 2/3 meter from the rotation axis; the 2kg mass is only 1/3 m from the axis. We are told that the rod slows from 20 rpm to 0 in 5s, its angular acceleration is:

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{(\omega_f - \omega_i)}{\Delta t} = \frac{(0 - 2.09\,\text{rad}/\text{s})}{5\,\text{s}} = -0.42\,\text{rad}\big/\text{s}^2$$

Now, it is simple to find the needed torque :

$$\tau = I\,\alpha \Rightarrow \tau = \left(0.67\,\text{kgm}^2\right)\left(0.42\,\text{rad}\big/\text{s}^2\right) = 0.28\,\text{N m}$$

and the torque must be directed opposite the initial rotation of the rod.

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7. Since the pivot is at the middle of the board, the board generates no net torque and we do not need to know its mass. If the pivot were not centered, we would need to take into account the center of mass. Objects to the left of the pivot will cause a ccw torque, masses to the right will generate a cw torque.

The 5 kg cat will generate a torque of 5 g (2 m); and the 2 kg bowl of tuna generates an opposite torque of 2 g (2 m).

Our torque balance becomes : $4\,g + 4\,g\,d = 10\,g$

The terms on the left are the torques to the left of the pivot (in terms of the acceleration of gravity, g) and the term on the right is the torque of the 5 kg cat. It is easy to show that:

$$d = \frac{3}{2}\,\text{m}$$

so the cat must stand 1.5 m to the left of the pivot. This is a ridiculous scenario. Anyone who has lived with cats knows they are not going to balance on the board with 2kg of tuna nearby.

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8. This is a problem in conservation of energy including rotational energy. We did not really get to these in class, so this type of problem will not appear on the exam.

We solve this using standard conservation of energy techniques, but in this case we have to include rotational kinetic energy.

At the top, the sphere has potential energy of M g h where h is the vertical distance above the bottom. At the bottom, it has converted all of its PE into KE, but there are now two forms of KE, translational and rotational. The translational KE is simply 1/2 M v^2 where v is its linear velocity at the bottom, and the rotational KE is:

$$KE_{rot} = \frac{1}{2} I \omega^2$$

for a sphere, I = 2/5 M R^2, and (see text for detailed derivation) a point on the edge of the sphere has a linear velocity of v = ω R. Therefore, we can write the rotational energy as:

$$KR_{rot} = \frac{1}{2}\left(\frac{2}{5} M R^2\right)\left(\frac{v^2}{R^2}\right) = \frac{1}{5} M v^2$$

Our energy balance becomes:

$$M g h = \frac{1}{2} M v^2 + \frac{1}{5} M v^2 = \frac{7}{10} M v^2$$

$$\text{or } v = \sqrt{\frac{10}{7} g h}$$

Recall that a particle sliding down a plane reaches a speed of $\sqrt{2 g h}$ after falling through a height h. The sphere is slower because some of the initial PE went into rotation so less PE transformed into linear speed. Using the numbers in the problem, h = 2.1 sin 25 = 0.89m, so:

$$v = \sqrt{\frac{10}{7}\left(9.8 \text{ m}/\text{s}^2\right)(0.89 \text{ m})} = 3.52 \text{ m}/\text{s}$$

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9. In this problem, we find the total moment of inertial by summing the individual moments. The rod is pivoted through its center, so has a moment of inertia of :

$$I_{rod} = \frac{1}{12} M L^2$$

The mass of m_1 is a distance L/2 from the pivot, so its moment of inertia is:

$$I_1 = M\left(\frac{L}{2}\right)^2 = \frac{M L^2}{4}$$

The mass m_2 is L/2 from the pivot, so

$$I_2 = M\left(\frac{L}{4}\right)^2 = \frac{M L^2}{16}$$

and the total moment is the sum of these three contributions.

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10. This is a problem dealing with the conservation of angular momentum. You will not be responsible for this matieral on the final. Since the sand is falling vertically, it exerts no torque on the disk, and thus angular momentum of the sand/disk system is conserved. This means that :

$$I_o \omega_o = I_f \omega_f$$

where the subscripts “o” and “f” refer to initial and final states respectively. The initial angular momentum of the disk is 1/2 M R^2. The final moment of inertia is larger because of a ring of sand of mass M/2 at a distance of r from the rotation axis. The moment of inertia due to the sand is m r^2, so the final moment of inertia is:

$$I_{\text{final}} = \frac{1}{2} M R^2 + \frac{M}{2} r^2$$

The conservation of angular momentum becomes:

$$\frac{1}{2} M R^2 \omega_o = \left(\frac{1}{2} M R^2 + \frac{M}{2} r^2\right) \omega_f \Rightarrow$$

$$\omega_f = \frac{\frac{1}{2} M R^2 \omega_o}{\frac{1}{2} M R^2 + \frac{M}{2} r^2} = \frac{\omega_o}{1 + \left(\frac{r}{R}\right)^2}$$