PHYS 111 K
HOMEWORK #1-- SOLUTIONS

1. Consider the equation :

$$u^{2/3} - 5\,u^{1/3} + 6 \;=\; 0$$

Notice that if we make the substitution, $z = u^{1/3}$, we can rewrite our equation as :

$$z^2 - 5\,z + 6 \;= 0$$

which is a trivial quadratic equation to solve. You might have noticed that the original equation is a quadratic in $u^{1/3}$ (since $u^{2/3}$is the square of the middle term $u^{1/3}$) . Thus, the z equation has simple answers of z = 2, 3. Since :

$$z \;=\; u^{1/3} \;\Rightarrow\; u \;=\; z^3$$

our solution to this equation are u = 2^3, 3^3, or u = 8, 27.

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2. The first step in this problem is to find a relationship between the decay constant λ and the half life (which we will designate at $t_{1/2}$). Since we know that half the remaining sample decays in a time equal to the half life, we know that the value of $N(t) \;=\; 1/2\, N_o$ when t = $t_{1/2}$, so we can write:

$$\frac{1}{2}\,N_o \;=\; N_0\,e^{-\lambda\,t_{1/2}}$$

Dividing by common factors and taking the ln of both sides, we get :

$$\ln(1/2) \;=\; -\,\lambda\,t_{1/2} \;\Rightarrow\; \lambda \;=\; \frac{\ln 2}{t_{1/2}}$$

In the case of C - 14, the value of λ is :

$$\lambda \;=\; \frac{\ln 2}{5730\text{ yrs} * \pi\,10^7\text{ s/yr}} \;=\; 3.85\times 10^{-12}\,s^{-1}$$

where I have made use of the astronomers' mnemonic that 1 year $\approx \pi \cdot 10^7$ s.

Now, when 90 % of the sample has decayed, the value of N (t) = 0.1 N_o, and

$$0.1\,N_o \;=\; N_o\,e^{-\,3.85\,10^{-12}\,t}$$

(notice that the argument of the exponential is dimensionless). Dividing common factors and taking ln both sides :

$$t \;=\; \frac{-\ln 0.1}{3.85\times 10^{-12}\,s^{-1}} \;=\; 5.98\times 10^{11}\,s \;=\; 19\,047\text{ yrs.}$$

Now, how might be estimate whether this answer makes any sense or not? We know that 1/2 the sample remains after 1 half life, 1/4 remains after 2 half lives, and 1/8 will remain after three half lives. In other words, we expect 87.5 % of the original material to have decayed after 3 half lives. So we should be able to conclude that our answer (when 90 % has decayed) should be a bit more than three half lives. Since one half life is 5730 yrs, three half lives are 17190 years, so an answer on the order of 19000 yrs seems reasonable.

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3. For this problem it is helpful to remember the small angle approximations :

$$\tan\theta \approx \theta \approx \sin\theta \quad \text{when } \theta \text{ is measured in radians.}$$

As viewed from the Earth, the "upper" half of each object will subtend an angle given by :

$$\tan\theta = \frac{\text{object radius}}{\text{distance from Earth}}$$

So we have :

$$\tan\theta_{\text{sun}} \approx \theta_{\text{sun}} = \frac{7\times10^5\ \text{km}}{1.5\times10^8\ \text{km}} = 0.0047\ \text{radians} = 0.27^{\text{o}} = 963\,''$$

$$\tan\theta_{\text{moon}} \approx \theta_{\text{moon}} = \frac{1736\ \text{km}}{4\times10^5\ \text{km}} = 0.0043\ \text{radians} = 0.25^{\text{o}} = 895\,''$$

Remember that the numbers computed above are the half diameters of the objects, so that the apparent angular diameters of the moon and sun are roughly 1/2 degree. These results show that the sun and the moon have very similar apparent angular diameters when viewed from the Earth, a confluence of geometric factors that allows for the stunning astronomical phenomenon of total solar eclipses. Note also that the moon' s orbit is not circular, so the apparent size of the moon can vary by almost 10 % throughout the year.

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4. In order for the range to equal the maximum height, these two expressions should be equal, so that :

$$\frac{2\,v^2\sin\theta\cos\theta}{g} = \frac{v^2\sin^2\theta}{2\,g}$$

Dividing common factors :

$$2\cos\theta = \frac{\sin\theta}{2} \Rightarrow \tan\theta = 4 \text{ or } \theta = 76^{\text{o}}$$

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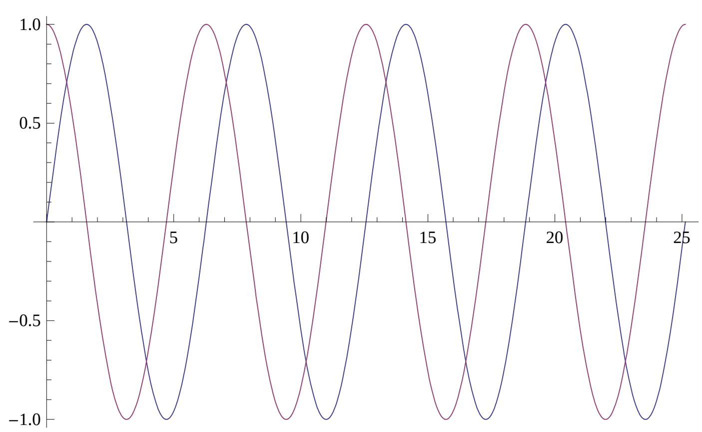
5. Let' s look at the curves of sin x and cos x over four cycles :



Recall from trig class that sin x = 0 when x = 0, π, 2 π, 3 π, etc. Thus, the expression sin (k π/2) will be zero whenever (k π/2) is an integer, which occurs whenever k is even. The expression cos (n π) = 0 whenever n is 1/2, 3/2, 5/2, so there are no integral values of n that will cause cos (n π) to be zero. (Note: I actually wanted this to read cos(n π/2) but...typo...)