

PHYS 111K

HOMEWORK #2

Due : 8 Sept. 2015

All answers must show complete solutions. Make sure you use proper units during each stage of the calculation. Consider for example a question asking you to compute the acceleration of an object that starts from rest and reaches a speed of 30 m/s in 3 s, your answer should look like :

$$a = \frac{v_f - v_0}{t} = \frac{30 \text{ m/s} - 0 \text{ m/s}}{3 \text{ s}} = 30 \text{ m/s}^2$$

An answer consisting of

$$a = 30/10 = 3 \text{ m/s}^2$$

would lose credit both for lack of a complete solution (not showing the relevant equation you are solving) and also for lack of units throughout.

1. If a particle is traveling along the positive x axis, can its acceleration vector point along the negative x axis? Explain your answer.

Solution : Let's consider the simple pictorial below :



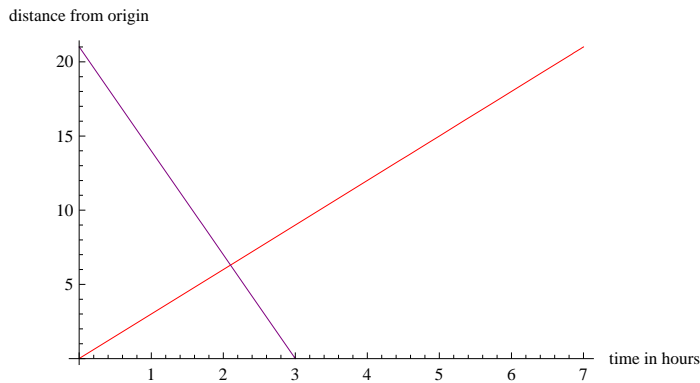
This depicts the motion of an object to the right; the v_2 arrow is clearly shorter than the v_1 arrow. The definition of acceleration is $v_2 - v_1$, so it is clear that the direction of the acceleration arrow in this case is to the left, so that it is possible for the acceleration vector to point in the direction opposite to motion.

2. This problem is sometimes called the 'Monk and the Monastery' problem in mathematical logic. A monk leaves his office in a city one day exactly at noon and walks at a constant speed to the monastery located a distance D away on the outskirts of town. Because it was a pleasant day, he walked slowly and arrived at the monastery at 7 pm. The next day, he leaves the monastery at noon and walks toward his office, following exactly the same route he took the day before. However, because there is an impending storm, he walks much faster (but at a constant speed) and arrives at his office at 3 pm.

Is there any point along the path that he passes at the same time each day? Explain the reasoning behind your answer. Construct a position vs. time graph, and plot each day's motion on the same graph. Use this graph to support your answer above. Finally, write the relevant equations of motion and determine the time and location of this point, or show mathematically that such a point does not

exist.

Solution: The graph below should help in analyzing this situation. In order to plot a graph, I needed to specify numbers, so arbitrarily set the distance from the city to the monastery as 21 km (since that divides nicely by 7 and 3). The red line shows the outbound trip; the purple line shows the inbound trip. Indeed there is exactly one point of intersection. Logicians approach the problem this way. Imagine there are two travelers, one in the city and one in the monastery, and they each set out exactly at noon traveling the same path. No matter what their relative speeds, they must intersect at exactly one point. To solve for the time and place of intersection, write the equations of these lines (or write the basic kinematic equations) and solve simultaneously.



The outbound line has the equation :

$x = (D/7) t$ since $D/7$ is the slope of this line (and thus the speed of the outbound traveler).

The inbound line has the equation :

$$x = D - (D/3) t$$

since D is the y - intercept and $- D/3$ is the slope (speed). Equating these two equations produces :

$$\frac{D}{7} t = D - \frac{D}{3} t \Rightarrow t \left(\frac{D}{7} + \frac{D}{3} \right) = D \Rightarrow \frac{10}{21} t = 1 \Rightarrow t = \frac{21}{10}$$

In other words, the intersection will occur at $t = 2.1$ hrs

3. Consider a river with parallel banks. A boat can travel at a speed of v in still water travels in a river where the current flows always in the same direction and always at the speed V . The boat travels a distance D downstream (in the direction of the current), reverses direction and travels back to its starting point. Assuming no time elapses in reversing direction, show that the time to complete the roundtrip is :

$$t = \frac{2 D}{v^2 - V^2}$$

What is the time needed to complete a trip if $v = V$. Explain why you obtain this result.

Solution : The best way to approach this problem is to divide into the downstream portion and the return (upstream) portion. The only equation we need to use is the basic relationship between

distance speed and time in one dimensional constant motion, namely :

$$\text{dist} = \text{speed} * \text{time} \Rightarrow \text{time} = \frac{\text{dist}}{\text{speed}}$$

Going downstream, the current speed V adds to your own speed of v , so the total speed downstream is $v + V$, and the time needed to go downstream is :

$$t_{\text{down}} = \frac{D}{v + V}$$

Returning, the speed with respect to the river banks is now $v - V$, and the return time is :

$$t_{\text{up}} = \frac{D}{v - V}$$

the total time is the sum of these two, or

$$t_{\text{total}} = t_{\text{down}} + t_{\text{up}} = \frac{D}{v + V} + \frac{D}{v - V} = \frac{D(v - V) + D(v + V)}{(v + V)(v - V)} = \frac{2Dv}{v^2 - V^2}$$

If $v = V$ you will make no progress against the current and never return to your starting position, as reflected in the infinite time predicted by the equation when the denominator goes to zero.

4. A rock is dropped from rest from a cliff of height H above a well. After a time T elapses (from the moment when the rock was dropped) the sound of the rock splashing into the water is heard by an observer at the top of the cliff. If the (constant) speed of sound is c , derive an expression for the height of the cliff H in terms of T , c , and g (the acceleration due to gravity). (We will neglect all effects due to friction). Now, use this expression to determine the height of the cliff is the sound is heard 9.37 s after being dropped. Assume $c = 343$ m/s

Solution : As in the problem above, you should divide this into two phases : the first determines how long the rock is in free fall, the second determines the time for the sound wave to return. We have :

$$T = t_1 + t_2$$

where T is the total time (given as 9.37 s), t_1 is the time of free fall, and t_2 is the time for the sound wave to travel the distance H .

For objects starting from rest, the distance traveled is related to the time of free fall by :

$$H = \frac{1}{2} g t_1^2 \Rightarrow t_1 = \sqrt{2H/g}$$

Sound travels at a constant speed c so will travel a distance H in a time :

$$t_2 = \frac{H}{c}$$

Thus, we have that the total time is :

$$T = \sqrt{2H/g} + \frac{H}{c}$$

You can solve this as a quadratic in \sqrt{H} or rewrite the T equation as:

$$\left(T - \frac{H}{c}\right) = \sqrt{2H/g}$$

Squaring both sides yields :

$$T^2 - 2\frac{TH}{c} + \frac{H^2}{c^2} = 2H/g$$

Combining terms :

$$\frac{H^2}{c^2} - 2\left(\frac{T}{c} + \frac{1}{g}\right)H + T^2 = 0$$

This is a quadratic equation in H, whose solution is :

$$H = \frac{2\left(\frac{T}{c} + \frac{1}{g}\right) \pm \sqrt{4\left(\frac{T}{c} + \frac{1}{g}\right)^2 - \frac{4T^2}{c^2}}}{(2/c^2)}$$

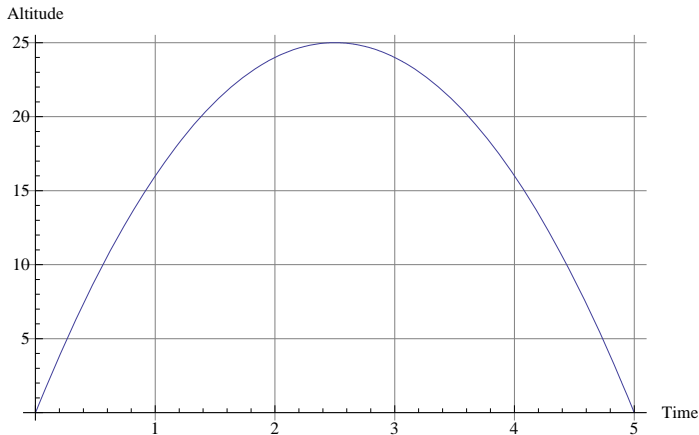
When you substitute values and solve for H, you will get two solutions. The positive branch of the square root yields an answer of 30, 069 meters. The negative branch yields a solution of 343 meters. How do we determine which is the meaningful solution?

Approach it like this : suppose the sound wave took no time at all to return, then the upper limit of the height of the cliff would be how far an object can fall in 9.37 seconds. This distance is

$$\text{distance} = \frac{1}{2} g t^2 = \frac{1}{2} (9.81 \text{ m/s}^2) (9.37 \text{ s})^2 = 430 \text{ m}$$

The cliff cannot be 30 km tall, thus 343 m is the meaningful answer.

5. You are the leader of a scientific expedition to a recently discovered exoplanet. One of your experiments involves firing a projectile vertically upward. Below is the graph of the height of the projectile as a function of time. $t = 0$ corresponds to the launch. From these data, determine the free fall acceleration on the planet and the initial speed of your projectile. Since there is no atmosphere on the planet, you are safe to ignore any effects of air friction. Time is measured in seconds; altitude in meters.



Solution : We want to find two pieces of information, local acceleration due to gravity (let's still call it g) and initial launch velocity. We know the object reached a maximum height (H) of 25 m in a time of 2.5 s. We will need two independent equations to make use of these two observations. Let's consider the projectile's motion from launch to apex; we know at apex (the highest point) its instantaneous velocity is zero, thus, we can write for this portion of the trip :

$$g = \frac{v_f - v_o}{t} \quad (1)$$

We also know the relationship between initial velocity, final velocity, acceleration and distance traveled :

$$v_f^2 = v_o^2 - 2 g H \text{ or since } v_f = 0, v_o^2 = 2 g H \quad (2)$$

We can rewrite eq. (1) as :

$$v_o^2 = g^2 t^2$$

and substitute into eq. (2) to find the magnitude of g :

$$g^2 t^2 = 2 g H \Rightarrow g = 2 H / t^2 = \frac{2 * 25 \text{ m}}{(2.5 \text{ s})^2} = -8 \text{ m/s}^2$$

Since g acts down, its value is negative. Knowing the value of g , we can find the initial velocity easily from eq. (1) :

$$v_o = - g t = -(-8 \text{ m/s})(2.5 \text{ s}) = 20 \text{ m/s}$$