PHYS 111 K
HOMEWORK #7
Solutions

1. Block A has a mass M and rests on an incline of angle θ. Block B has a mass of m and rests on an incline of angle ϕ. We will consider forces acting either along the plane or perpendicular to the plane, and will call up the plane as the positive direction in each case.

a) If the planes are frictionless, the only forces acting along the plane are the tension in the rope and the component of gravity acting down the plane. Since the rope is massless, the tension is the same everywhere in the rope. Since the two blocks move as a system, they must have the same magnitude of acceleration. Writing Newton's second law for each block :

$\Sigma F_A = T - M g \sin\theta = - M a$ (the sign is negative since A slides down the surface)

$\Sigma F_B = T - m g \sin\phi = m a$

Subtratcing the B equation from the A equation gives :

$$- M g \sin\theta + m g \sin\phi = - (M + m) a$$

$$a = \frac{(M \sin\theta - m \sin\phi) g}{M + m}$$

b) Now we have to consider the effects of friction. Since friction opposes motion, the force of friction will act up the plane for block A, and down the plane for B. The friction force is μ N where μ is the coefficient of friction and N is the normal force between the block and the plane.

On an inclined plane, the component of gravity perpendicular to the plane is g cos θ, so we can write Newton's second law as :

$$\Sigma F_A = T - M g \sin\theta + \mu M g \cos\theta = -M a$$

$$\Sigma F_B = T - m g \sin\phi - \mu m g \cos\phi = m a$$

If you subtract equations and solve for a, you will obtain :

$$a = \frac{(M \sin\theta - m \sin\phi) g - \mu g (M \cos\theta + m \cos\phi)}{m + M}$$

Note that this expression yields the answer in part a) if $\mu = 0$ as expected.

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2. Let's begin by writing Newton's second Law for the 20 kg block (let's call its mass m). In the horizontal direction, the only forces acting are tension and friction, and in the vertical direction, weight and the normal force. Since we are being asked to find the conditions under which the block does not accelerate, we set the sum of forces to zero :

$$\Sigma F_{hor} = T - f = T - \mu N = 0$$

$$\Sigma_{F_{\text{vert}}} = N - m\,g = 0 \Rightarrow N = m\,g$$

We can easily combine these equations to find that $T = \mu\,m\,g$ (where μ is the coefficient of static friction).

Now we have to find an expression for the tension in the rope (let's call it rope 1) that is connected to m. We will have to do this in steps, starting with finding the tension in the rope connected to mass A (let's call this rope 2). If the system is in equilibrium, then the tension in that rope must equal the weight of A. Next, consider the junction of the three ropes. We can apply Newton's second law to any part of this system, and this enables us to recognize that the sum of forces acting on this junction is zero. This means that the vertical component of the slanting rope must equal the weight of block A. If we call this rope 3, we can write :

$$T_3 \sin 45 = M_A\,g \Rightarrow T_3 = \frac{M_A\,g}{\sin 45}$$

The horizontal component of tension in rope 3 must then equal the horizontal component of tension in rope 1 (which is what we wanted to figure out once we started), and the horizontal component of tension in rope 3 is :

$$T_3\,(\text{horizontal}) = T_3 \cos 45 = \frac{M_A\,g}{\sin 45} \cdot \cos 45 = M_A\,g$$

The last step occurs because $\sin 45 = \cos 45$.

Now we combine results. We know that the tension in rope 1 is :

$$T_1 = \mu\,m\,g$$

and this must equal the horizontal component of tension in rope 3, or :

$$T_1 = T_3\,(\text{horizontal}) \Rightarrow \mu\,m\,g = M_A\,g \Rightarrow M_A = \mu\,m = 0.6 \cdot 20\,\text{kg} = 12\,\text{kg}$$

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3. Here we need to recognize that if the astronaut exerts a force on the satellite, the satellite will exert the same magnitude of force on the astronaut. The objects exert a force on each other only during the time of contact; once they separate there are no longer forces acting on them and they will travel at the speed they achieved at the moment of separation.

If the astronaut exerts a 100 N force on the 640 kg satellite, the satellite achieves an acceleration of

$$a_{\text{sat}} = \frac{100\,\text{N}}{640\,\text{kg}} = 0.16\,\text{m}\,\text{s}^{-2}$$

In the 0.5 s of contact, the satellite (starting from rest) will achieve a speed of :

$$v_{\text{sat}} = a_{\text{sat}}\,t = 0.16\,\text{m}\,\text{s}^{-2} \cdot 0.5\,\text{s} = 0.08\,\text{m}\,/\,\text{s}$$

The satellite exerts the same force on the 80 kg person whose acceleration is

$$a_{\text{person}} = \frac{100\,\text{N}}{80\,\text{kg}} = 1.25\,\text{m}\,\text{s}^{-2}$$

and the person will reach a velocity of 0.62 m/s during the time of contact. The relative velocity of the two objects is then 0.70 m/s, and after a minute they will be 0.70 m/s · 60 s = 42 m apart.

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4. We are told that the force at one end of the cable is 100 N. If the cable were massless, then we would know that the tension in the cable would be 100 N throughout. However, this is a massive cable so the tension at the other end will be less than 100 N.

Let' s analyze the situation by applying Newton' s second law to the 20 kg block. If the block is moving across a frictionless surface, the only force acting on it is the tension from the cable. We are told that the block reaches a speed of 4 m/s in 2 s, indicating an acceleration of 2 m/s^2. If a 20 kg block accelerates at this rate, the force acting on it must be 40N (from F = m a). Thus, the difference in tension across the ends of the cable is 60N (what is the mass of the cable?).

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5. We will need to be careful about units here. If the 140 g (= 0.14 kg) ball slows from 30 m/s to 0 in 0.0015 s, the acceleration of the ball is :

$$a = \frac{0 \, m/s - 30 \, m/s}{0.0015 \, s} = -20,000 \, m \, s^{-2}$$

and the force acting on the ball is F = m a = -2800 N. By Newton' s third law, if this is the force your face exerts on the ball, this is also the force the ball exerts on your face. Using the data in the problem, your forehead is safe, but your cheekbone will fracture.

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6. We will apply Newton' s second law to each mass (we cannot assume the system is moving at constant speed). Let' s call the larger mass M and the smaller mass m, and set down as the positive direction.

There is a component of gravity acting down the plane, the force of friction acting up the plane, and the tension in the rope. The tension acts up the plane on the 2 kg block, but down the plane for the 1 kg block. The blocks are conected so have the same acceleration, and we must be careful to use different coefficients of friction for the two blocks. Thus, the laws of motion become :

$$\Sigma F_M = M \, g \sin \theta - \mu_M \, M \, g \cos \theta - T = M \, a$$

$$\Sigma F_m = m \, g \sin \theta - \mu_m \, m \, g \cos \theta + T = m \, a$$

I think it will be easier to eliminate T first and solve for acceleration, then use the value of a to find T. Adding equations this time, we get :

$$(M + m) \, g \sin \theta - (\mu_M \, M + \mu_m \, m) \, g \cos \theta = (M + m) \, a$$

or

$$a = \frac{(M + m) \, g \sin \theta - (\mu_M \, M + \mu_m \, m) \, g \cos \theta}{M + m}$$

Using the numbers given :

$$a = \frac{g[(3 \, kg \sin 20) - (0.1 \cdot 2 \, kg + 0.2 \cdot 1 \, kg) \cos 20]}{3 \, kg} = 2.1 \, m/s^2$$

Using this value for the acceleration in either of the equations above will yield the tension in the

string. Using the equation for M we get :

$$T = M g \sin\theta - \mu_M M g \cos\theta - M a = 2\,\text{kg}\left[9.8\,\text{m}/\text{s}^2\,(\sin 20 - 0.1\cos 20) - 2.1\,\text{m}/\text{s}^2\right] = 0.66\,\text{N}$$

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7. Call the rope to the right rope 1 and the other rope 2. Down will be negative; to the right will be positive. The blocks are moving as a system, so will all have the same magnitude of acceleration. Newton' s second law for each mass then becomes :

$$\Sigma F_{3\,\text{kg}} = T_1 - 3\,g = -3\,a \tag{1}$$

$$\Sigma F_{2\,\text{kg}} = T_1 - T_2 - 2\,\mu\,g = 2\,a \tag{2}$$

$$\Sigma F_{1\,\text{kg}} = T_2 - g = a \tag{3}$$

Subtract eq. (3) from eq. (1) to get an expression for T_1 - T_2 that we can use later :

$$T_1 - T_2 - 2\,g = -4\,a \quad \text{or} \quad T_1 - T_2 = 2\,g - 4\,a$$

Use this result to replace the tensions in eq. (2) :

$$2\,g - 4\,a - 2\,\mu\,g = 2\,a \quad \text{or} \quad 6\,a = 2\,g\,(1-\mu)$$

Therefore, $a = \dfrac{1}{3}\,g\,(1-\mu) = \dfrac{g\,(1-0.3)}{3} = 0.23\,g$