PHYS 111 K
HOMEWORK #8

Solutions

1. A chain of uniform density and length L lies on a table with a portion of the chain overhanging the edge of the table. If the coefficient of friction between the chain and the table is μ, what is the maximum fraction of the chain that can overhang the edge before the chain slides off the table?

Solution : Let' s call the total mass of the chain M. Let' s call the length of chain overhanging the table x; then the length of chain on the table is L - x. The mass of the overhanging portion is M (x/L), and the mass of the chain lying on the table is M - M (x/L) = M (1 - (x/L))

The force of gravity acting on the overhanging portion is M (x/L) g. This will be the force that pulls the chain on the table. If the system does not accelerate, then the force of friction acting on the chain (which is on the table) will equal the weight of the overhanging portion. In other words :

$$\mu\, M \left(1 - \frac{x}{L}\right) g \;=\; M \left(\frac{x}{L}\right) g$$

or solving for the fraction of the chain hanging over the table, x/L :

$$\frac{x}{L} \;=\; \frac{\mu}{1 + \mu}$$

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2. Conceptual question #2 on the top of p. 210.

Solution : When the car is at the bottom of this valley, we can think of the car as moving along the bottom of a vertical circle. This means the car is accelerating (even if it is moving at constant speed); the forces acting on the car, the normal force and its weight combine to produce a centripetal acceleration acting toward the center of the vertical circle. When the car is at the bottom of the circle, the direction of the centripetal force is up (toward the center of the circle), and our force law is :

$$\Sigma F_r \;=\; N \,-\, m\,g \;=\; +\,\frac{m\,v^2}{r}$$

This shows that the resultant vector acts up (and that the normal force has greater magnitude than the weight). Of the five options given to you, (a) is correct. Option (b) shows the normal force equal to the weight, and option (c) indicates weight is greater than the normal force (which would require the net force to act down). The last two options include horizontal forces.

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3. Problem 2 on the bottom of p. 210.

Solution : The horizontal and vertical motions are independent of each other. I assume the rocket is pointed upward and the thrust is vertical. Therefore, we have no forces in the x direction, and the rocket' s horizontal motion will be a constant 3 m/s. In the vertical direction, there is both a force of gravity acting down and the thrust of the rocket acting up, so we have in the vertical direction :

$$a_y = \frac{F_{thrust} - m\,g}{m} = \frac{8\text{ N} - 4.9\text{ N}}{0.5\text{ kg}} = 6.2\text{ m}/\text{s}^2\text{ (upward)}$$

We can compute how long it will take the rocket to reach a height of 20 m :

$$y(t) = y_o + v_{oy}\,t + \frac{1}{2}\,a\,t^2$$

The initial height and vertical velocities are zero (the rocket accelerates from v = 0 due to the thrust), so we have that the time to reach a height H (as long as there is fuel to produce the 8 N of thrust) is :

$$t = \sqrt{2\,\frac{H}{a}} = \sqrt{2 \cdot 20\text{ m}/6.2\text{ m}/\text{s}^2} = 2.54\text{ s}$$

Since the rocket has a horizontal speed of 3 m/s, it should be launched 7.62 m ahead of the circle.

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4. Problem 12, p. 211

Solution : The Earth (mass m) is attracted to the sun (mass M) by :

$$F = \frac{G\,m\,M}{r^2} = m\,a \Rightarrow a = \frac{G\,M}{r^2}$$

where G is the Newtonian gravitational constant and r is the Earth - sun distance. Substituting numbers :

$$a = \frac{\left(6.67\times 10^{-11}\text{ N m}^2\text{ kg}^{-2}\right)\left(2\times 10^{30}\text{ kg}\right)}{\left(1.5\times 10^{11}\text{ m}\right)^2} = 0.006\text{ m s}^{-2}$$

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5. Problem 32, p. 212

Solution: The equations we need here are derived using the techniques developed for the challenge example on pp. 207 - 208 of the text. The text derives the minimum speed the car can have without sliding down the ramp. To find the maximum speed, we have to realize that the force of friction must act down the plane to prevent the car from sliding up. This allows us to write Newton' s second law in the radial and vertical directions as :

$$\Sigma F_{radial} = N \sin\theta + f \cos\theta = \frac{m\,v^2}{r}$$

The friction term is positive because friction acts down the plane, and the radial component of this vector is toward the center of the circle. In the vertical direction :

$$\Sigma F_{vertical} = N \cos\theta - f \sin\theta - mg = 0$$

Setting f = μ N in both equations, we get (with some slight rewriting) :

$$N\sin\theta + \mu N\cos\theta = \frac{m v^2}{r} \tag{1}$$

$$N\cos\theta - \mu N\sin\theta = m g \tag{2}$$

If we divide equation (1) by equation (2) we can cancel out common factors of N and m and get :

$$\frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta} = \frac{v^2}{r g} \Rightarrow v_{max} = \sqrt{r g\left(\frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta}\right)}$$

Using the parameters given in the problem (r = 70 m, θ = 15 degree, μ for rubber on concrete (see table p. 149) = 1), we get :

$$v_{max} = 34.5\,m/s.$$

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6. Problem 43, p. 212

Solution : At the bottom of the ride, Newton' s second law tells us that :

$$N - m g = \frac{m v^2}{r} \Rightarrow N = m\left(g + \frac{v^2}{r}\right)$$

(Make sure you understand the sign on each term in this equation.) We are told the values for m and r (and already know g). To find v, we make use of the information that the wheel completes one revolution every 4.5 s, so :

$$v = \frac{2\pi r}{P} = \frac{2\pi(8\,m)}{4.5\,s} = 11.1\,m/s$$

so the normal force experienced at the bottom of the circle is :

$$N = 55\,kg\left(9.8\,m/s^2 + \frac{(11.1\,m/s)^2}{8\,m}\right) = 1396\,N$$

At the top of the ride, the centripetal force points down to the center of the circle so Newton' s second law becomes :

$$N - m g = \frac{-m v^2}{r} \Rightarrow N = m\left(g - \frac{v^2}{r}\right) = 55\,kg\left(9.8\,m/s^2 - \frac{(11.1\,m/s)^2}{8\,m}\right) = 308\,N$$

Riders will remain on the wheel as long as N > 0. We can find the speed for which N = 0 by setting :

$$g - \frac{v^2}{r} = 0 \Rightarrow v = \sqrt{r g} = \sqrt{8\,m * 9.8\,m/s^2} = 8.85\,m/s$$

The question asks us to find the period of the wheel that has this speed, so :

$$P = \frac{2\pi r}{v} = \frac{2\pi(8\,m)}{8.85\,m/s} = 5.7\,s$$

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7. Problem 48, p. 213

Solution : Assuming the string is massless (and we have to since we are not given a mass for the string) we know the tension in the string is the same throughout. For the hanging mass to be at rest, the tension in the string must equal its weight, or T = m_2 g. The tension in the string must also provide the centripetal force on mass m_1, so that T also equals $m_1 v^2$/r. Equating these two expressions for the same tension, we get:

$$m_2\, g = \frac{m_1\, v^2}{r} \Rightarrow v = \sqrt{\frac{m_2}{m_1}\, g\, r}$$

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8. A newly discovered planet has a radius that is 1/2 the Earth' s radius and a mass that is 1/10 the mass of the Earth. What is the value of surface gravity on this planet (it is much easier to solve this using ratios).

Solution : We derive the expression for surface gravity by realizing weight is the force of gravity of a planet acting on an object, so we have for an object on the surface of a planet of radius R :

$$F_{grav} = \frac{G\, m\, M}{R^2} = m\, g \Rightarrow g = \frac{G\, M}{R^2}$$

where m is the mass of the object, M is the planetary mass, R is the planetary radius and G is the Newtonian grav constant. If we take the ratio of g on planet X to g on the Earth we get :

$$\frac{g_X}{g_{Earth}} = \frac{\left(\frac{G\, M_x}{R_x^2}\right)}{\left(\frac{G\, M_{earth}}{R_{Earth}^2}\right)} = \frac{M_x}{M_{Earth}} \cdot \left(\frac{R_{Earth}}{R_X}\right)^2 = \frac{1}{10} \cdot 2^2 = \frac{4}{10}$$

The surface gravity on the newly discovered planet is 4/10 the value on the Earth, or roughly 4 m/s^2.