PHYS 111 K
HOMEWORK #9-- Solutions

1. This is an interesting problem in that it requires us to solve several different sub - problems before getting to the final answer. Another aspect of this problem is that we have to distinguish between tangential acceleration (due to the thrust of the rocket) and radial acceleration (due to the tension in the string.) This is a problem where we can save ourselves a lot of effort and computation by using symbols until the very end.

One key to the problem is finding out how long it will take for the tension in the string to exceed the tensile strength of 50 N. We know that the tension in the string must provide the centripetal force causing the rocket to rotate. We also know that the rocket will experience a tangential acceleration due to the constant thrust force perpendicular to its motion.

Let' s start by computing how long it will take for the tension in the rope to exceed 50 N. The linear tangential acceleration of the rocket is described by Newton' s Second Law :

The tension in the tube will provide the centripetal accelration needed for the rocket to move in its circular path. In other words,

$$T \; = \; \frac{m \, v^2}{r}$$

where r is the radius of the tube. Since the rocket starts from rest and accelerates (we' ll determine the acceleration in a bit below) we know its tangential velocity varies according to

$$v\,(t) \; = \; v_o + a\,t \; = \; a\,t \,(\text{since } v_o \; = \; 0).$$

Thus, we can determine the time it will take for the tension in the tube to reach a certain value (in this case, the tensile strength of the tube or 50 N) from :

$$T \; = \; \frac{m\,(a\,t)^2}{r} \; \Rightarrow \; t^2 \; = \; \frac{r\,T}{m\,a^2}$$

But we are not done yet; we are asked to find how many revolutions the rocket will make before the tension reaches 50N. For this, we need to determine how far the rocket travels in this time, and from that distance we will find how many revolutions the rocket makes. Recall the equation of motion:

$$s = s_o + v_o t + \frac{1}{2} a t^2$$

where s is the distance traveled, s_o and v_o are the initial positions and velocities, a is the acceleration and t is the time of travel. In our case, the initial values are both zero, so we have simply

$$s = \frac{1}{2} a t^2$$

so we know that the rocket will travel a distance :

$$s = \frac{1}{2} a t^2 = \frac{1}{2} a \left(\frac{r\,T}{m\,a^2} \right) = \frac{1}{2} \frac{r\,T}{m\,a}$$

before the tube breaks. Since one revolution corresponds to the circumference of the orbit ($2\,\pi\,r$), the number of revolutions the rocket makes is then :

$$\sharp\,\text{revs} = \frac{s}{2\,\pi\,r} = \frac{1}{2\,\pi\,r} \cdot \frac{1}{2} \frac{r\,T}{m\,a} = \frac{1}{4\,\pi} \frac{T}{m\,a} \tag{1}$$

Notice that this result is independent of the radius of the orbit. We are given the tensile strengh T = 50 N and m = 0.5 kg, we now must use Newton' s second law to find an expression for a.

We begin with the familiar :

$$\Sigma \mathbf{F} = m\,\mathbf{a}$$

Here, the tangential forces acting on the rocket are the constant thrust force of 4 N and the friction between the steel rocket and steel table (if you did not catch that the author wanted you to include friction, I will not deduct points if you do the rest of the calculation correctly). Thus, the linear tangential acceleration of the rocket is :

$$a = \frac{F - \mu\,m\,g}{m} \frac{4\,\text{N} - 0.6 \cdot 0.5\,\text{kg} \cdot 9.8\,\text{m}/\text{s}^2}{0.5\,\text{kg}} = 2.1\,\text{m}/\text{s}^2$$

where F is the 4 N thurst and μ is the coefficient of kinetic friction of steel on dry steel (= 0.6 per table on p. 149). And now, we are in a position to compute the number of revolutions made by the rocket before the tube snaps. We substitute T = 50N, a = 2.1 m/s^2 and m = 0.5 kg into eq. (1) and obtain finally:

$$\sharp\,\text{revs} = \frac{1}{4\,\pi} \frac{T}{m\,a} = \frac{1}{4\,\pi} \frac{50\,\text{N}}{0.5\,\text{kg} \cdot 2.1\,\text{m}/\text{s}^2} = 3.8\,\text{revs}$$

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2. Consider the following (not quite to scale) diagram :
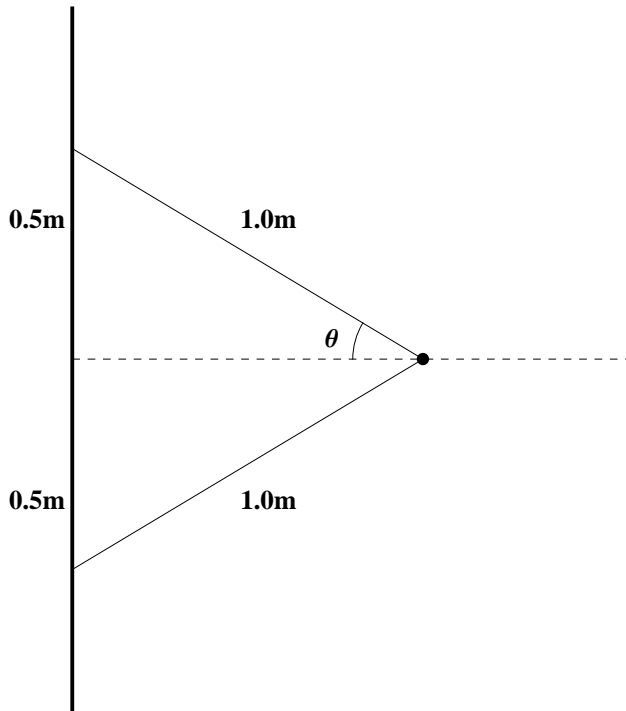
The forces acting on the mass (represented by the black dot) are the tensions in the ropes and the weight of the mass. We will use a standary x - y coordinate axis; positive x will be toward the center of rotation (to the left) and positive y is up. Note that the ropes make the same angle with the radial direction (represented by the dashed line). We write Newton' s second law in the x and y directions :

$$\Sigma F_x \,=\, T_1 \cos\theta \,+\, T_2 \cos\theta \,=\, \frac{m\,v^2}{r}$$

$$\Sigma F_y \,=\, T_1 \sin\theta \,-\, T_2 \sin\theta \,-\, m\,g \,=\, 0$$

I am calling the tension in the upper rope T_1 and the tension in the lower rope T_2. The horizontal forces must combine to produce the centripetal force $m\,v^2/r$, and note carefully that the radius of the orbit r is NOT the same as the length of the ropes; we will have to compute the value of r using the Pythagorean theorem. Rewriting these equations we get:

$$(T_1 + T_2)\cos\theta \,=\, \frac{m\,v^2}{r}$$

$$(T_1 - T_2)\sin\theta \,=\, m\,g$$

We can use the distances given in the diagram to determine quickly that $\sin\theta = 1/2$ and $\cos\theta = \sqrt{3}/2$ (this will also be the value of r, make sure you recognize why). We now have numerical values for all parameters except our unknowns, the tensions in the ropes. Rewriting we obtain:

$$(T_1 + T_2) = \frac{m\,v^2}{r\cos\theta} = \frac{0.3\,\text{kg}\,(7.5\,\text{m/s})^2}{\frac{\sqrt{3}}{2}\cdot\frac{\sqrt{3}}{2}} = 22.5\,\text{N}$$

$$(T_1 - T_2) = \frac{m\,g}{\sin\theta} = \frac{0.3\,\text{kg}\cdot 9.8\,\text{m/s}^2}{0.5} = 5.9\,\text{N}$$

Adding equations yields $2\,T_1 = 28.4$N so that $T_1 = 14.2$N. From this it is easy to show that $T_2 = 8.3$N

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3. The impulse is $\int F(t)\,dt$ or the area under the curve shown in fig. ex 9.4. There are three triangles; the two smaller ones will contribute negative values of impulse, and the large one will be positive. The two smaller triangles have a base of 2 ms (0.002s) and a height of -500N, so the area of each of the smaller triangles is - 0.5N s. The total impulse due to the smaller triangles is then -1.0 Ns. The large triangle has a base of 0.006s and a height of 2000N, so its area is 6Ns, and the total impulse for the entire interval is 5 Ns.

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4. The impulse acting on the object is F Δt = - 8 N (0.5 s) = -4 N s. The impulse momentum theorem tells us :

$$F\,\Delta t = \Delta\,(m\,v)$$

so we know the change in momentum of the object due to this force will be - 4 N s. Using the definition of momentum :

$$\Delta\,(m\,v) = m\,v_f - m\,v_i = -4\,\text{Ns} \Rightarrow v_f = \frac{m\,v_i - 4\,\text{N s}}{m} = \frac{2\,\text{kg}\cdot 1\,\text{m/s} - 4\,\text{N s}}{2\,\text{kg}} = -1\,\text{m/s}$$

$$\text{Note}: 1\,\text{N s} = 1\,\text{kg m/s}. \text{ The object is now moving to the left at } 1\,\text{m/s}.$$

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5. This is a problem involving the conservation of momentum. The exploding object was initially at rest, so its initial momentum was zero. Since the forces causing the explosion were all internal to the system, there are no external forces acting and the momentum of the system is conserved. This means the vector sum of the three momenta must be zero. The graph shows two momenta :

$$\mathbf{p_1} = 3\,\hat{\mathbf{x}} + 0\,\hat{\mathbf{y}}$$
$$\mathbf{p_2} = -2\,\hat{\mathbf{x}} + 2\,\hat{\mathbf{y}}$$

In order for the total momentum to be zero, the third fragment must have components :

$$\mathbf{p_3} = -1\,\hat{\mathbf{x}} - 2\,\hat{\mathbf{y}}$$

If you sum these three vectors, you will see they sum to zero as is required by the law of conservation of momentum.

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6. Let' s do this in the observer' s frame of reference (i.e., yours and mine) and convert the answer to the squid' s frame at the end. As we see it, the squid is drifting at 0.4 m/s so has an initial momentum of 1.5 kg $\cdot$ 0.4 m/s = 0.6 kgm/s. The force of expelling the water is internal to the squid/water system, so conservation of momentum holds, and we can write

$$\text{Initial momentum} = 0.6\,\text{kgm/s} = \text{final momentum} = 2.5\,\text{m/s}\cdot 1.4\,\text{kg} - 0.1\,\text{kg}\cdot V$$

Remember that the mass of the squid decreases by 0.1 kg to emit the fuel (water). We can solve for

V, the velocity of the water in our reference frame and obtain :

$$V = \frac{0.6 \, \text{kgm}/\text{s} - 2.5 \cdot 1.4 \, \text{kgm}/\text{s}}{0.1 \, \text{kg}} = -29 \, \text{m}/\text{s}$$

In the frame of reference of the squid (moving forward at 2.5 m/s), the water appears to be expelled at - 31.5 m/s (or - 32 m/s if you are a sig fig purist as apparently is the author).

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7. The simplest way to approach this problem is to use the conservation of momentum. Since the push - off is internal to the system consisting of the two skaters (and since we are assuming ice is frictionless even though the problem does not state so explicitly), we can write :

$$\text{momentum before push} = \text{momentum after push}.$$

Since the skaters were stationary before the push, their total momentum was zero, so that after the push we have :

$$m \, \mathbf{v} + M \, \mathbf{V} = 0 \Rightarrow m \, v = - M \, V$$

where upper case letters refer to the heavier skater and lower case letters to the lighter one. Using the values given, we can conclude that the lighter skater' s velocity was :

$$50 \, v = -75 \, V \Rightarrow v = \frac{-3}{2} \, V$$

or 1.5 times greater than the 75 kg skater' s. Since they traveled the same distance (they started in the center of a circular rink), the 50 kg skater must have arrived in 2/3 of the time of the 75 kg skater, or 13.3 s.