PHYS 111
HOMEWORK #10

Due : 15 Nov. 2016

1. Problem 75, p. 218

Solution : The key points here are to recognize this as a conservation of energy problem and also that there is no friction in the system. The latter means that the total mechanical energy (potential + kinetic) is constant throughout the car' s journey. If the car starts from rest at point A, its total energy is in the form of gravitational potential energy.

The diagram is sufficiently complicated and you might feel the urge to calculate speeds at many different points along the trip. However, all we have to do is determine from the diagram that the car has descended a vertical distance of 13 m in going from A to B. Therefore, the kinetic energy at B must equal the loss in potential energy between A and B, and we have :

$$\frac{1}{2} m v_B^2 = m g (h_A - h_B)$$

or

$$v_B = \sqrt{2 g (h_A - h_B)} = \sqrt{2 \cdot 9.8 \text{ m}/\text{s}^2 \cdot 13 \text{ m}} = 16 \text{ m}/\text{s}$$

At B, the track exerts a normal force on the car that acts down; gravity acts down, and these two forces must combine to produce the centripetal force that acts toward the center of the loop (which is also down). Setting down as the positive direction, Newton' s second law gives us :

$$N + m g = \frac{m v^2}{r} \Rightarrow N = m\left(\frac{v^2}{r} - g\right) = 350 \text{ kg}\left(\frac{256 \text{ m}^2/\text{s}^2}{6 \text{ m}} - 9.8 \text{ m}/\text{s}^2\right) = 11\,500 \text{ N}$$

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2. Use the diagram for problem 81 on p. 218 of the text. The car starts from rest at point A, which is a height h above the bottom of the track. The radius of the loop is r.

a) Draw a FBD for the car when it is at the top of the circular loop. (5)

Solution : At the top of the track, the normal force and gravity act down, and these combine to produce the centripetal force which also acts down.

b) What is the minimum speed needed to keep the car from falling off the track? (10)

Solution : From above, we can write the second law as (let' s call down the positive direction) :

$$N + m g = \frac{m v^2}{r}$$

In order to stay in contact with the track, the normal force must be positive. The minimum speed to accomplish this occurs when N = 0, or when

$$m g = \frac{m v^2}{r} \Rightarrow v^2 = r g$$

c) What is the height h required so that the car achieves this speed at the top of the loop? (10)

Solution : We use conservation of energy methods to determine the height from which the car must start. At point A, the car has zero kinetic energy and potential energy equal to m g h. At point B, the car has potential energy equal to 2 m g r (the height above the ground at B is 2r), and kinetic energy equal to 1/2 m v_b^2. Equating these gives us:

$$0 + m g h = \frac{1}{2} m v_b^2 + 2 m g r$$

However, we know from above that the speed at B satisfies :

$$v^2 = r g$$

so energy conservation gives us:

$$m g h = \frac{1}{2} m g r + 2 m g r \Rightarrow h = \frac{5}{2} r$$

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3. Problem 82, p. 218.

Solution : We first compute the speed of the block just before it reaches the section with friction. If the object drops from rest through a vertical distance of h, its speed will be :

$$v = \sqrt{2 g h}$$

where h is 4m in this problem. We use kinematics to find the distance the object travels before coming to rest:

$$v_f^2 = v_0^2 + 2 a s$$

In thi case, we know the final velocity is zero, the initial velocity is given above (i.e., the speed of the object when it reaches the friction section). We need to compute a in order to find s. We can find a since we know the force acting on the object is friction. On a level plane, we know that:

$$f_k = \mu_k N = \mu_k m g = m a \Rightarrow a = \mu_k g$$

Putting all these together, we obtain:

$$0 = (2 g h) + 2(-\mu_k g) s \Rightarrow s = \frac{h}{\mu_k} = \frac{4 m}{0.2} = 20 m$$

The easiest way to determine the work done by friction is to apply the work energy theorem. The block lost all of its energy to friction; therefore, since the work done by a force equals the change in kinetic energy, we know the work done by friction must equal the KE of the object before moving across the friction surface. Since conservation of energy tells us that the KE equals the initial PE, we know the work done by friction must equal the initial PE, or - m g h = 2kg · g · 4m = - 78.4J

Let' s verify this by using the equation for work. The total work is :

$$W = f_k \cdot s \cos\theta$$

where f_k is the work done by friction, s is the distance traveled, and θ is the angle between the force and displacement, which here is 180^o so that $\cos\theta = -1$. The force due to friction is:

$$f_k = \mu_k N = \mu_k m g = 0.2 \cdot 2\,kg \cdot 9.8\,m/s^2 = 3.92\,m/s^2$$

$$W = 3.92\,m/s^2 \cdot 20\,m\,(-1) = -78.4\,J$$

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4. A rubber bullet of mass m and speed v strikes a can and bounces backward. A steel jacketed bullet of mass m and speed v strikes the same can and moves through the can, emerging from the opposite with a speed v/2. In which case was more force exerted on the can? Justify your answer.

Solution : Here we have to remember that momentum and velocity are vectors. Let' s call the initial direction of the bullet' s motion the positive direction. In the case of the rubber bullet, the final velocity is in the opposite direction, so the total change in momentum is :

$$\Delta(m\,\mathbf{v}) = m(\mathbf{v}_{final} - \mathbf{v}_{initial})$$

The magnitude of this quantity will be greater than the magnitude of the initial momentum, because the bullet had to reverse its direction, and the force for changing the momentum comes from the interaction with the can.

In the second case, the change in momentum is much less, since the final and initial velocities are in the same direction:

$$\Delta(m\,\mathbf{v}) = m\left(\frac{v_o}{2} - v_o\right) = \frac{-m\,v_o}{2}$$

Thus, it is most likely that the force exerted by the rubber bullet is greater.

5. Problem 8, p. 249

a) ***Solution***: Assuming the skaters start with no kinetic energy, we can solve part a) by using the conservation of momentum. Our system consists of the two skaters. Before the push, they have no momentum; in other words, the total momentum of the two skater system is zero. Therefore, the system will have zero momentum after the push. Conservation of momentum tells us :

$$0 = m_1 v_{1A} + m_2 v_{2A}$$

where the subscript A means after collision, and the subscripts 1 and 2 refer to the two skaters. We know the speed of one skater, therefore:

$$m_1 v_{1A} = -m_2 v_{2A} \Rightarrow v_{2A} = -\left(\frac{m_1}{m_2}\right) v_{1A} = -\frac{74\ \mathrm{kg}}{63.8\ \mathrm{kg}}(1.5\ \mathrm{m/s}) = -1.74\ \mathrm{m/s}$$

where the negative sign means the motion is opposite the motion of the heavier skater. Since there is no kinetic energy at the beginning, the kinetic energy after the pushing is:

$$KE_A = \frac{1}{2} m_1 v_{1A}^2 + \frac{1}{2} m_2 v_{2A}^2 = 0.5 \cdot (74\ \mathrm{kg})(1.5\ \mathrm{m/s})^2 + \frac{1}{2} \cdot 63.8\ \mathrm{kg}\,(1.74\ \mathrm{m/s})^2 = 180\ \mathrm{J}$$

This kinetic energy derives from the work done by the skaters in pushing away from each other. A force (exerted on each skater) moved through a distance; this amount of work equals the increase in KE.

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6. Problem 13, p. 249

Solution : This problem combines concepts of momentum conservation and energy conservation. Our system consists of the two blocks and the spring. Since there are no external forces acting on this system, we can apply the conservation of momentum.

The system has no momentum when the spring is compressed; the elastic force of the spring is internal to the system, so even after the spring is released, the total momentum of the system is zero. After the blocks are in motion, we can write :

$$0 = m_A v_A + m_B v_B$$

Using the data given in the text, it is easy to show that:

$$v_A = \frac{-m_B}{m_A} v_A = -\frac{3\ \mathrm{kg}}{1\ \mathrm{kg}} \cdot 1.2\ \mathrm{m/s} = -3.6\ \mathrm{m/s}$$

Since there is no friction between the blocks and the surface, the final kinetic energy must equal the initial elastic potential energy, or

$$U_{el} = \frac{1}{2} k (\Delta x)^2 = \frac{1}{2}\left(m_A v_A^2 + m_B v_B^2\right) = \frac{1}{2}\left(1\ \mathrm{kg} \cdot (3.6\ \mathrm{m/s})^2 + 3\ \mathrm{kg} \cdot (1.2\ \mathrm{m/s})^2\right) = 8.64\ \mathrm{J}$$