PHYS 111
HOMEWORK #13

Due : 3 pm Friday, 9 Dec. 2016

This is an optional homework assignment. If you wish to receive credit for this assignment, turn it in to my office by 3 pm this Friday. Your score on this homework will replace your lowest non-zero homework score (this will not replace a homework you did not turn in or for a homework where you received a zero for academic dishonesty.) Since this material will be covered on the final, I urge all students to attempt these problems, and study carefully the solutions which will be posted at 3 pm on Friday.

1. A billiard ball moving across a pool table at a speed of 1.5 m/s makes a head on collision with an identical ball. Find the speed of each ball after collision if : a) the second ball is initially at rest; b) the second ball is moving toward the first ball at 1 m/s; c) the second ball is moving away from the first ball at 1 m/s. (5 pts each part). Assume all collisions are elastic.

Solution : In the case the target ball is at rest, we can use the equations from class and the text (8.11 and 8.12) :

$$V_{A,f} = \frac{m_A - m_B}{m_A + m_B} V_{A,i} = 0; \; V_{B,f} = 2\frac{m_A}{m_A + m_B} v_{A,i} = V_{A,i} = 1.5\,\mathrm{m/s}$$

In the other two cases, we cannot use these equations since they are derived explicitly assuming the second ball is stationary. Rather, we have to write out the equations of momentum and energy conservation and solve them explicitly. In the first case, where the two balls approach each other, the conservation of momentum is written :

$$1.5\,m - m = m\,v_{A,f} + m\,v_{B,f}$$

Since the masses are the same, I have not bothered to use subscripts; also, I am assuming the direction of the ball moving at 1.5 m/s is the positive direction. We can cancel out common factors of mass and write this more simply as:

$$0.5\,\mathrm{m/s} = v_{A,f} + v_{B,f} \tag{1}$$

The conservation of energy is written:

$$\frac{1}{2}m\,(1.5\,\mathrm{m/s})^2 + \frac{1}{2}m\,(-1\,\mathrm{m/s})^2 = \frac{1}{2}m\,v_{A,f}^2 + \frac{1}{2}m\,v_{B,f}^2$$

or, after cancelling common factors and evaluating terms :

$$3.25\,\mathrm{m^2/s^2} = v_{A,f}^2 + v_{B,f}^2 \tag{2}$$

We can rewrite eq. (1) as $v_{B,f}$= 0.5 – $v_{A,f}$ and use this in eq. (2)

$$3.25\ \mathrm{m}^2 \big/ \mathrm{s}^2 \ = \ \mathrm{v}_{\mathrm{A,f}}^2 \ + \left(0.5 - \mathrm{v}_{\mathrm{A,f}}^2\right)$$

which yields the quadratic equation:

$$3.25\ \mathrm{m}^2 \big/ \mathrm{s}^2 \ = \ 2\ \mathrm{v}_{\mathrm{A,f}}^2 \ - \mathrm{v}_{\mathrm{A,f}} \ + 0.25$$

solving this quadratic yields the answers $v_{A,f}$ = -1 m/s or +1.5m/s. If $v_{A,f}$ = - 1m/s, $v_{B,f}$= +1.5m/s. If $v_{A,f}$ = +1.5m/s, then $v_{B,f}$= - 1m/s. The geometry of the situation makes it clear that the balls will not continue on in their current directions; ball A rebounds at -1m/s, and ball B rebounds to move in the positive direction at +1.5m/s.

In the second case, where the second ball moves at the same speed as the first ball, our mode of analysis is the same except that the conservation of momentum equation becomes (after dividing out common terms):

$$1.5\ \mathrm{m/s} \ + \ 1\ \mathrm{m/s} \ = \ \mathrm{v}_{\mathrm{A,f}} \ + \ \mathrm{v}_{\mathrm{B,f}}$$

or

$$\mathrm{v}_{\mathrm{A,f}} \ + \mathrm{v}_{\mathrm{B,f}} \ = \ 2.5\ \mathrm{m/s}$$

The conservation of energy equation becomes :

$$3.25\ \mathrm{m}^2 \big/ \mathrm{s}^2 \ = \ \mathrm{v}_{\mathrm{A,f}}^2 \ + \ (2.5\ \mathrm{m/s} \ - \ \mathrm{v}_{\mathrm{A,f}})^2$$

when this quadratic is expanded and solved, we find that $v_{A,f}$ = 1m/s and $v_{B,f}$ = 1.5 m/s.

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2. Squids propel themselves by ingesting water and squirting it out (essentially a jet propulsion mechanism.) Suppose a 100 kg giant squid (they actually grow larger than this) is initially at rest in the ocean. It ingests 10 kg of water which it then expels backward at a speed of 10 m/s. What will be the forward speed of the squid?

Solution : This is a conservation of momentum problem. Initially, the squid/water system is at rest and has no momentum. After the water is thrust, the total momentum of the system must continue to be zero. We have then :

$$0 \ = \ -10\ \mathrm{kg} \ \cdot \ 10\ \mathrm{m/s} \ + \ 100\ \mathrm{kg}\ \mathrm{v}_{\mathrm{squid}}$$

this yields a speed of the squid of 1 m/s in the direction opposite the expelled water.

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3. Consider a merry go round of radius 3 m whose moment of inertia around its rotational axis is 100 kg m^2 . While the merry go round is at rest, a child of mass 30 kg runs tangential to the outer edge at a speed of 5 m/s and then jumps on. a) What is the moment of inertia of the merry go round/child system once the child has jumped on? b) What is the angular velocity of the system once the child has jumped on? (10 pts each part).

Solution : This is a conservation of angular momentum problem. A necessary step is to compute the moment of inertia of the combined child/wheel system. We are told the moment of inertia of the wheel; the child can be considered a point mass located on the edge of the wheel. Since we know the contribution to the moment of inertia of a point mass is m r^2 where r is the distance of the mass from the rotation axis, the total moment of inertia of this system is:

$$I_{total} = I_{wheel} + I_{child} = 100\ kg \cdot m^2 + (30\ kg)(3\ m)^2 = 370\ kg\ m^2$$

Since there are no torques external to the child/wheel system, angular momentum is conserved. Before the child hops on to the wheel, the total angular momentum of the system is

$$L_{Child} = m_{child}\, v_{child}\, r = 30\ kg \cdot 5\ m/s \cdot 3\ m = 450\ kg\ m^2/s$$

This must equal the total angular momentum after the child is on the wheel:

$$L_{wheel/child} = I\,\omega = 370\ kg\ m^2\ \omega = 450\ kg\ m^2/s \Rightarrow \omega = 1.22\ rad/s$$

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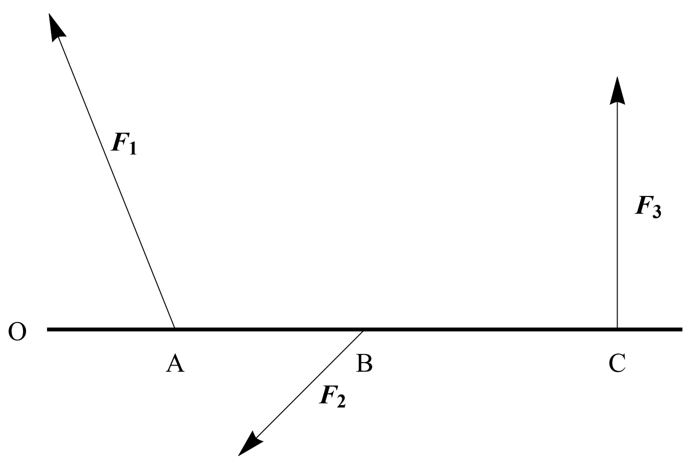
4. Consider the diagram below :



Three forces act on a plank as shown above. $\boldsymbol{F_1}$ has a magnitude of 40N, $\boldsymbol{F_2}$ has a magnitude of 20N, and $\boldsymbol{F_3}$ has a magnitude of 30N. The distances of A, B and C from O are, respectively, 2m, 5m and 9m. $\boldsymbol{F_1}$ and $\boldsymbol{F_2}$ make angles of 45^o with respect to the plank. Find the torques generated by each force around O, and the direction and magnitude of total torque.

Solution : Forces 1 and 3 will generate positive (counterclockwise) torques around O; force 2 will generate a clockwise torque. We can find the magnitudes of each torque from the equation :

$$\tau = r\,F \sin\phi$$

For each force we have:

$$\tau_1 = +(2\ m)(40\ N) \sin 45 = 56.6\ Nm$$

$$\tau_2 = -(5\ m)(20\ N) \sin 45 = -70.7\ N$$

$$\tau_3 = +(9\ m)(30\ N) \sin 90 = +270\ Nm$$

The total torque is in the positive (counterclockwise sense) and equals 256Nm

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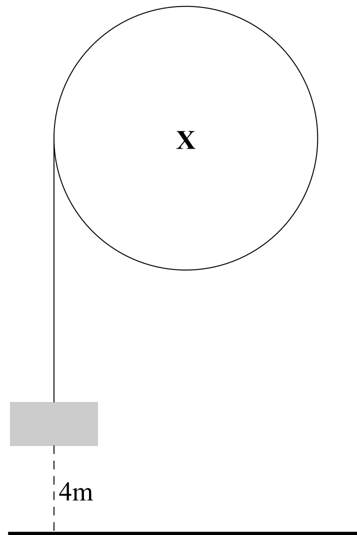
5. Use the diagram for question 12 on p. 314. Consider a mass m hanging from a string attached to a solid cylindrical wheel of mass M and radius R. If the system is released from rest, find the accleration of the stone, the tension in the wire, and the angular acceleration of the wheel.

Solution : Consider the diagram below :



The rotation axis runs through the "X" perpendicular to the page. For this problem, we use Newton's second law for forces and torques. The forces acting on the mass m are the tension in the rope and the weight of m:

$$\Sigma F_m = T - m g = - m a$$

The tension in the rope causes the torque on the wheel; the rope acts at a distance R from the rotation axis, so the torque due to the tension is simply TR. Then we have:

$$\Sigma\tau = T R = I \alpha$$

where I is the moment of inertia of the wheel (= 1/2 M R^2) and α is the angular acceleration of the wheel. Since the rope pulls without slipping, α is related to a and R via α = a/R. Combining these results, we can write the torque equation as:

$$T R = \frac{1}{2} M R^2 \left(\frac{a}{R}\right) \Rightarrow T = \frac{1}{2} M a$$

Using this result in the force equation gives us:

$$\frac{1}{2} M a - m g = - m a \Rightarrow a = \frac{m g}{M/2 + m}$$

The tension is then

$$T = \frac{1}{2} M a = \frac{1}{2} M \left(\frac{m g}{M/2 + m}\right) = \frac{m M g}{M + 2 m}$$

and the angular acceleration of the wheel is

$$\alpha = \frac{T\,R}{I} = \frac{\frac{m\,M\,g\,R}{M+2\,m}}{\frac{1}{2}\,M\,R^2} = \frac{2\,m\,g}{R\,(M+2\,m)}$$

6. Use energy methods to find the speed of the mass after it has fallen a height h.

Solution : Before the mass descends, it has GPE equivalent to m g h. When it has fallen a height h, all the GPE is converted into the translational energy of the mass and the rotational energy of the wheel :

$$m\,g\,h = \frac{1}{2}\,m\,v^2 + \frac{1}{2}\,I\,\omega^2 = \frac{1}{2}\,m\,v^2 + \frac{1}{2}\left(\frac{1}{2}\,M\,R^2\left(\frac{v}{R}\right)^2\right) = \frac{1}{2}\,v^2\left(m + \frac{M}{2}\right)$$

$$\Rightarrow v = \sqrt{\frac{2\,m\,g\,h}{(m + M/2)}}$$

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7. Problem 22, p. 314.

Solution : This is an angular momentum problem. Angular momentum is defined by :

$$L_\oplus = m_\oplus\,r\,v$$

where $m_\oplus$ is the mass of the Earth, r is the average value of the size of the Earth's orbit, and v is the average orbital velocity of the Earth. The orbital velocity of the Earth is

$$v = \frac{2\,\pi\,r}{P} = \frac{2\,\pi\left(1.5\times10^{11}\text{ m}\right)}{3.15\times10^{7}\text{ s}} = 3\times10^{4}\text{ m/s}$$

where P is the length of a year (and = $3.15\cdot10^7$s), so we have:

$$L_\oplus = \left(6\times10^{24}\text{ kg}\right)\left(3\times10^{4}\text{ m/s}\right)\left(1.5\times10^{11}\text{ m}\right) = 2.7\times10^{40}\text{ kg m}^2\big/\text{s}$$

The spin momentum of the Earth is:

$$L_\oplus = I\,\omega = \frac{2}{5}\,M\,R^2\,\omega$$

where M is the mass of the Earth, R is the radius, and ω is the angular velocity of the Earth and equals :

$$\omega = 2\,\pi/1\text{ day} = \frac{2\,\pi}{86\,400\text{ s}} = 7.3\times10^{-5}\text{ rad/s}$$

Therefore, the spin momentum of the Earth is :

$$L_\oplus = 0.4\left(6\times10^{24}\text{ kg}\right)\left(6.4\times10^{6}\text{ m}\right)^2\left(7.3\times10^{-5}\text{ rad/s}\right) = 7.2\times10^{33}\text{ kg m}^2\big/\text{s}$$

These results suggest it is valid to treat the Earth as particle in orbit around the sun; the radius of the Earth is much smaller than the orbital distance to the sun, and the spin momentum is much smaller than the orbital momentum.

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8. Problem 23, p. 314.

Solution : For each part of the problem, we will solve this by using :

$$L = m\,v\,r$$

where m is the mass of the bird, v is its velocity, and r is its distance from the pivot point. In this problem, only the distance from the pivot point varies. In part a), the angular momentum is in the positive (ccw) direction with a magnitude of

$$L_A = (0.3\text{ kg})\,(3.5\text{ m/s})\,(1.5\text{ m}) = 1.58\text{ kg m}^2\big/\text{s}$$

In part b), the bird is flying directly at the pivot point so will cause no torque if it hits there; since r = 0, the angular momentum is zero. In part c), the torque will be negative with magnitude:

$$L_C = 0.3\text{ kg}\cdot 3.5\text{ m/s}\cdot(1.8\text{ m} - 1.5\text{ m}) = 0.32\text{ kg m}^2\big/\text{s}$$

The last part is equivalent to part a) and yields the same result.

Now, consider this extension to the problem. Suppose instead of a bird the mass was a ball of putty (same mass, same speed) that will adhere when it hits the stick. In part a), when the pivot point (rotation axis) is at the top, we would expect the stick to begin to rotate with some angular speed. We can use the conservation of angular momentum to compute this rotational speed. Before the collision, the angular momentum of the putty/stick system is just the angular momentum of the putty :

$$L_{\text{before}} = m\,v\,r$$

When the putty clings to the stick, the moment of inertia of the stick/putty system becomes:

$$I_{\text{after}} = I_{\text{stick}} + I_{\text{putty}} = \frac{1}{3}M\,L^2 + m\,\lambda^2$$

where M is the mass of the stick, L is the length of the stick, m is the mass of the putty, and λ is the distance between the rotation axis and the position of the putty. The moment of inertia after collision is just the moment of inertia of the stick plus the contribution from the particle of mass m a distance λ from the axis. So, conservation of angular momentum tells us:

$$L_{\text{before}} = L_{\text{after}} \Rightarrow m\,v\,r = \left(\frac{1}{3}M\,L^2 + m\,\lambda^2\right)\omega_{\text{after}}$$

or ω_{after} =

$$\omega_{\text{after}} = \frac{m\,v\,r}{\frac{1}{3}M\,L^2 + m\,\lambda^2}$$

Study this extension carefully. You are likely to see this, or something very similar, again.

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9. Problem 29, p. 315.

Solution : Since the string is pulled in a direction perpendicular to the motion of the object, there is no external torque exerted and therefore no change in angular momentum. The angular momentum of an orbiting particle is :

$$\mathrm{L} = \mathrm{m\,v\,r} = \mathrm{m}\,(\omega\,\mathrm{r})\,\mathrm{r} = \mathrm{m}\,\omega\,\mathrm{r}^2$$

Therefore, since the total angular momentum does not change, we have :

$$\mathrm{m}\,\omega_{\mathrm{i}}\,\mathrm{r}_{\mathrm{i}}^2 = \mathrm{m}\,\omega_{\mathrm{f}}\,\mathrm{r}_{\mathrm{f}}^2 \Rightarrow \omega_{\mathrm{f}} = \omega_1 \left(\frac{\mathrm{r}_{\mathrm{i}}}{\mathrm{r}_{\mathrm{f}}}\right)^2 = 1.75\ \mathrm{rad/s} \left(\frac{0.3\ \mathrm{m}}{0.15\ \mathrm{m}}\right)^2 = 7\ \mathrm{rad/s}$$

We know that the linear velocity is $v = r\,\omega$, so that the change in kinetic energy is:

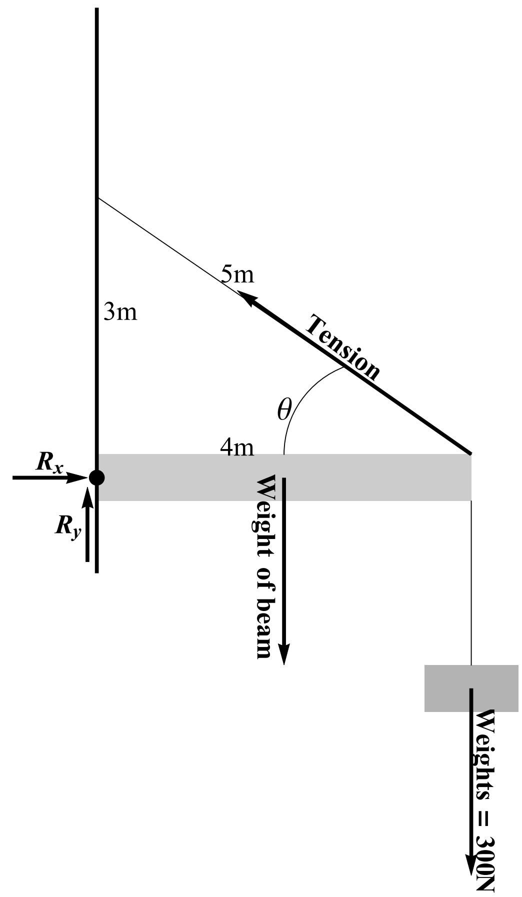
$$\Delta\,(\mathrm{KE}) = \frac{1}{2}\,\mathrm{m}\left(\mathrm{v}_{\mathrm{f}}^2 - \mathrm{v}_{\mathrm{i}}^2\right) =$$

$$\frac{1}{2}\,\mathrm{m}\left((\mathrm{r}_{\mathrm{f}}\,\omega_{\mathrm{f}})^2 - (\mathrm{r}_{\mathrm{i}}\,\omega_1)^2\right) = \frac{1}{2}\,(0.025\ \mathrm{kg})\left((0.15\ \mathrm{m}\cdot 7\ \mathrm{rad/s})^2 - (0.3\ \mathrm{m}\cdot 1.75\ \mathrm{rad/s})^2\right) = 0.01\ \mathrm{J}$$

By the work energy theorem, this is the work done to pull the string down.

10. Problem 41, p. 316

Solution : Consider the diagram below to help us identify forces : The beam and the weights each have a gravitational force acting down. The tension lies along the cable and has both an x and y component. The hinge exerts both a vertical and horizontal force on the beam (these are labelled R_y and R_x). The data given allow us to determine that that $\tan\theta = 3/4$, $\sin\theta = 3/5$ and $\cos\theta = 4/5$.



Writing the equilibrium conditions we get :

$$\Sigma F_x = R_x - T\cos\theta = 0 \Rightarrow R_x = T\cos\theta$$

$$\Sigma F_y = T\sin\theta + R_y - W_{beam} - W_{weights} = 0 \Rightarrow R_y + T\sin\theta = 450\,\mathrm{N}$$

Finally, we sum torques around the hinge; the torques due to the forces of the hinge are zero, so we have:

$$\Sigma\boldsymbol{\tau} = \text{torque due to tension} + \text{torque due to weight of beam} + \text{torque due to weights}$$

or

$$\Sigma\tau = T\sin\theta\,(L) - W_{beam}\,(L/2) - W_{weights}\,(L) = 0$$

Look carefully at the torque equation; we know the values of all the variables except the tension, so we solve for that first. Dividing out a common factor of L and substituting known values, we get:

$$0.6\,T - 0.5\,(150\,\mathrm{N}) - 300\,\mathrm{N} = 0 \Rightarrow T = \frac{375\,\mathrm{N}}{0.6} = 625\,\mathrm{N}$$

We can use the value of the tension in the force equations to determine the values of the reaction force of the hinge on the board. The x equation gives:

$$R_x = T\cos\theta = 0.8 \cdot 625\ \text{N} = 500\ \text{N}$$

The y equation gives:

$$R_y = 450\ \text{N} - 0.6\,(625\ \text{N}) = 75\ \text{N}$$

The total reaction force on the board has magnitude:

$$R_{\text{total}} = \sqrt{R_x^2 + R_y^2} = \sqrt{(500\ \text{N})^2 + (75\ \text{N})^2} = 506\ \text{N}$$

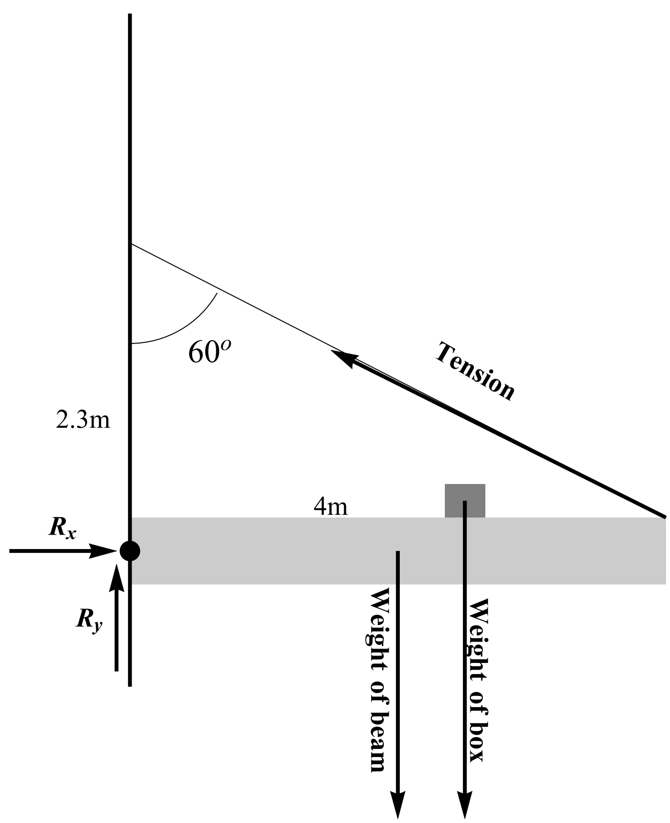
and the force acts in the direction given by:

$$\tan\phi = \frac{R_y}{R_x} = \frac{75\ \text{N}}{500\ \text{N}} \Rightarrow \phi = 8.5^{\text{o}}$$

This means that the reaction force of the hinge on the board acts in a direction that is 8.5^o above the positive x axis.

11. Problem 45, p. 316

Solution : The diagram below will allow us to identify forces and write the equations of equilibrium.



We begin by writing the equilibrium equations for forces and torques.

$$\Sigma F_x = R_x - T\sin 60 = 0 \Rightarrow R_x = T\sin 60$$

$$\Sigma F_y = R_y + T\cos 60 - W_{\text{beam}} - W_{\text{box}} = 0 \Rightarrow R_y = W_{\text{Beam}} + W_{\text{box}} - T\cos 60$$

Summing torques about the hinge allows us to set the torques due to R_x and R_y to zero, so:

$$\Sigma\tau = W_{\text{beam}}\left(\frac{L}{2}\right) + W_{\text{boz}}\left(\frac{5\,L}{8}\right) - T\sin 30\,(L) = 0$$

(the box is 5/8 of the beam length from the hinges).

We can divide out a common factor of L in the torque equation and solve for the tension in the cable:

$$T\sin 30 = 0.5\cdot 2500\,\text{N} + 0.625\cdot\,3500\,\text{N} \Rightarrow T = 6875\,\text{N}$$

Using this value of T in the ΣF_x equation yields

$$R_x = 6875\sin 60 = 5954\,\text{N}$$

and in the ΣF_y equation:

$$R_y = 6000\,N - 0.5\cdot 6875\,N = 2562\,N$$

The total reaction force of the hinge acting on the beam is

$$R_{\text{total}} = \sqrt{R_x^2 + R_y^2} = \sqrt{(5954\,\text{N})^2 + (2562\,\text{N})^2} = 6482\,\text{N}$$

and the reaction force makes an angle of

$$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right) = \tan^{-1}\left(\frac{2562}{5954}\right) = 23.2^{\text{o}}$$

The reaction force vector lies 23.2^o above the positive x axis as shown below.