PHYS 111
HOMEWORK #3

Due : 15 Sept. 2016

1. Let' s try it the right way this time. A person begins walking at noon one day from point A and travels at a constant velocity along a straight line path to point B a distance D away, arriving at 7 pm that evening. The very next day, the traveler leaves B at exactly noon, travels at a different constant velocity along exactly the same path, arriving at A at 3 pm.

Is there any point along the path that the traveler reached at the same time on both days? Justify your answer. If you conclude that there is such a time, determine the time at which this point was reached the location (distance from A) of this point.

Solution : There is exactly one point that the traveler reaches at the same time on consecutive days. Perhaps the clearest way to see this is to reframe the question. Instead of one traveler walking on two consecutive days, imagine there are two people walking on the same day, one leaving A at noon and one leaving B at the same time. If there is only one path they can take, they must pass each other at some point on the trail. This is the place that the traveler reaches at the same time on both days.

To quantify this, let's write the equation of motion for each day. Let's set x = 0 at point A. Then, x = D at point B. If the direction from A to B is positive, then velocity is positive on the first day, and negative on the second. The general equation of motion for one dimensional travel is:

$$x(t) = x_o + v_o t + \frac{1}{2} a t^2$$

Since there are no accelerations, we can ignore the last term. Since the traveler reaches A at 7pm on the first day, the velocity for that portion of the trip is D/7. On the second day, the velocity is negative (motion is in the opposite direction), and is equal to - D/3. This allows us to write:

$$x_A(t) = \frac{D}{7} t \qquad x_B(t) = D - \frac{D}{3} t$$

The person will be at the same point when $x_A = x_B$:

$$\frac{D}{7} t = D - \frac{D}{3} t$$

Combining terms:

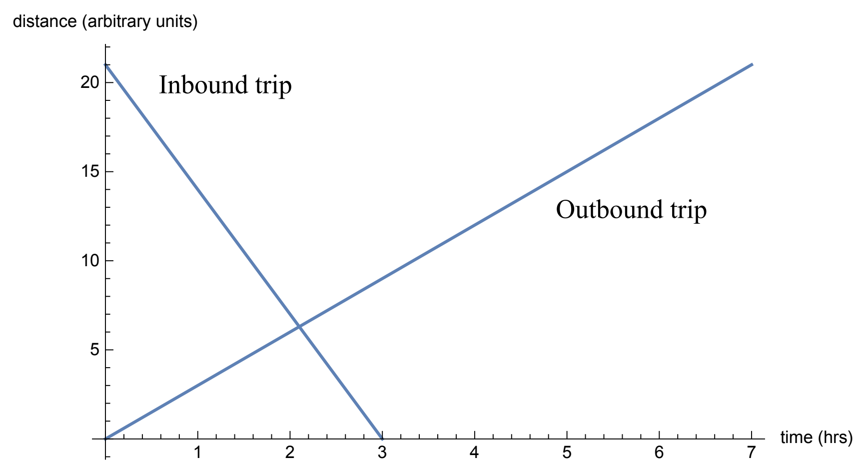
$$\left(\frac{D}{7} + \frac{D}{3}\right) t = D \Rightarrow t = \frac{1}{1/7 + 1/3} = 2.1 \text{ hrs}$$

The common point will be reached at 2:06 pm. We can find the distance from A using either equation of motion, using the equation for A:

$$x_A (t = 2.1) = \frac{D}{7} (2.1) = 0.3 D$$

This point is reached 30% of the way along the trail to B.

We can graph the scenario:



The graph should make clear that there is exactly one place occupied by the traveler at the same time on both days.

2. A student runs to catch the campus shuttle bus which is stopped at a red light. The student runs at a constant speed of 6 m/s (and can run no faster) to catch the bus. When the student is still 60 m from the (front door of) the bus, the bus begins to move and moves away at a constant acceleration of 0.18 m s^{-2}.

a) For how long and how far does the student have to run before reaching the bus? (10)

Solution : For this we have to construct the equations of motion for the person and the bus. The person runs at a constant rate; if we set the initial position of the person as x = 0, we can write the equation of motion as :

$$x_p (t) = 6 t$$

At t = 0, the bus is initially at rest and starts 60m ahead of the person. We are told it moves with a constant acceleration of 0.18 $m\, s^{-2}$. Since the initial velocity is zero, we can write the equation of motion of the bus as:

$$x_b(t) = 60 + \frac{1}{2} \cdot 0.18\, t^2 = 60 + 0.09\, t^2$$

The person will catch the bus when $x_p = x_b$, or when:

$$6\,t = 60 + 0.09\, t^2 \Rightarrow 0.09\, t^2 - 6\,t + 60 = 0$$

We recognize this as a quadratic equation in t, with the solutions:

$$t = \frac{6\,m/s \pm \sqrt{(6\,m/s)^2 - 4\,(60\,m)\left(0.09\,m/s^2\right)}}{2\left(0.09\,m/s^2\right)} = 12.2\,s,\ 54.4\,s$$

The person catches up to the bus at t = 12.2 s. If the person kept running past the bus, the bus would then catch up with the person at 54.4s (see graph below).

b) You should have found a second solution to the equation you solved for part a). What is the significance of the second solution? (Your graph from part d) might help you answer this question.) What is the speed of the bus at that time? (5)

Solution : The meaning of the second solution is noted above. Since the bus is moving with constant acceleration, its speed at any given time is described by :

$$a = \frac{\Delta v}{\Delta t} \Rightarrow \Delta v = a\,\Delta t \Rightarrow v_f - v_o = a\,(t_f - t_o)$$

In this case, the initial velocity (v_o) is zero, and if we set (as is the custom) the initial time to zero, this equation becomes simply:

$$v_f = a\,t \Rightarrow v\,(12.2\,s) = 12.2\,s \cdot 0.18\,m/s^2 = 2.2\,m/s$$

$$v\,(54.4\,s) = 54.4\,s \cdot 0.18\,m/s^2 = 9.8\,m/s.$$

c) If the student ran at a constant speed of 4 m/s, would the student catch the bus? (5)

If the person could run at a maximum constant speed of 4m/s, the relevant equation of motion would then be $x_p(t) = 4t$. Equating the position of the person to the bus would generate the quadratic equation:

$$0.09\, t^2 - 4\,t + 60 = 0$$

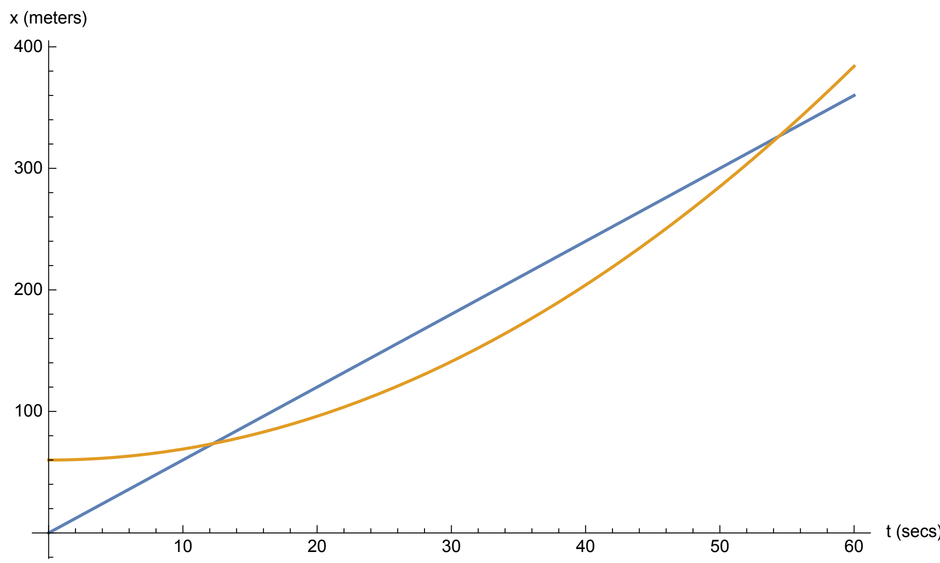
whose solution would require computing:

$$t = \frac{4\,m/s \pm \sqrt{(4\,m/s)^2 - 4\,(60\,m)\left(0.09\,m/s^2\right)}}{2\left(0.09\,m/s^2\right)}$$

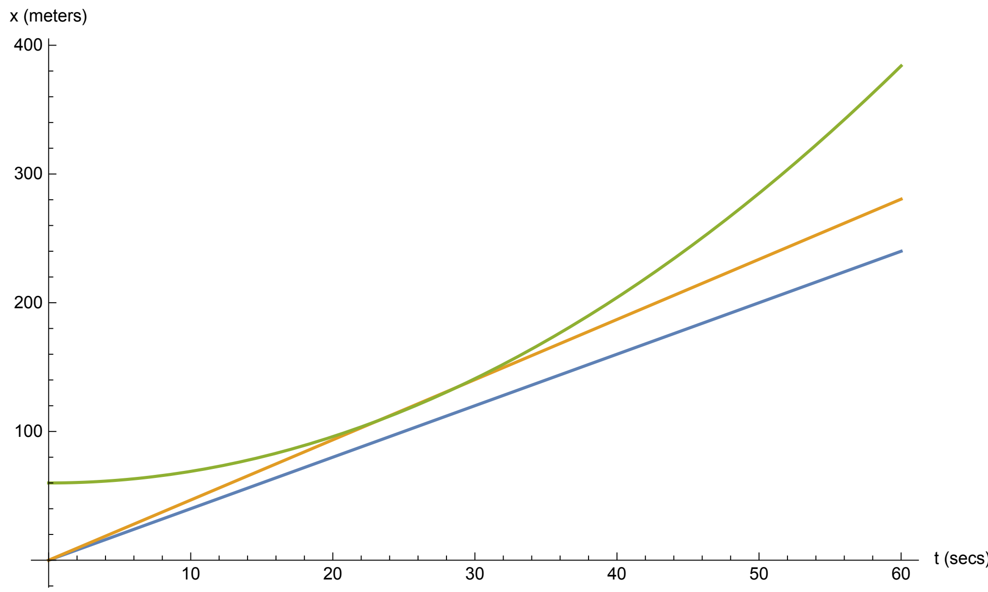
Let' s just focus on the discriminant (the stuff inside the square root). As long as the discriminant is positive or zero, we can compute a real (as opposed to imaginary) value for the terms in the square root, and thus compute real values for time. However, if the discriminant is negative, we obtain an imaginary result which tells us that the scenario is not physically significant. If the person' s speed is 4 m/s, the discriminant is negative with a value of - 5.6 m^2/s^2, showing us that running at 4 m/s will never catch this bus.

d) Use a real piece of graph (or quadrille) paper and draw the graphs of the motion of the bus and student on the same set of axes. Make sure axes are labeled and appropriate numerical values are placed on the axes. (5)

Solution : The first graph below shows the motion of the bus (the curved line) and the motion of the person running at a constant velocity of 6 m/s. Note that the bus starts at x = 60 m when t = 0, and that the person catches up to the bus at t = 12.2 s, but the bus finally overtakes the person at t = 54.4 s.



In the graph below, the motion of the bus (green curve) is depicted along with the hypothetical motions of a person running at 4 m/s (the blue line) and a person running at the minimum speed needed to catch the bus (the orange line). As calculated in part e), the minimum speed needed to just catch the bus is 4.65m/s.



Notice that the blue line never intersects the (green) curve depicting the bus' motion. Notice that the orange line is tangent to the bus curve at the point of intersection.

e) What is the minimum constant speed the student could have and still catch the bus? How long would the student have to run at this speed to catch the bus. (10)

We showed in part c) that running at 4m/s would not allow the person to reach the bus. But what is the minimum speed you could run? The minimum speed occurs when the value of the discriminat is zero, or when:

$$b^2 - 4\,a\,c = 0 \Rightarrow b^2 = 4\,a\,c = 4\,(60\,\text{m})\left(0.09\,\text{m}/\text{s}^2\right) \Rightarrow v = 4.65\,\text{m}/\text{s}$$

3. A ball is thrown vertically down from a building of height H with an initial speed of v. At the same time, an identical ball is thrown vertically upward from the same location with an initial speed of v. Which ball will hit the ground with the greater speed? Justify your answer either using logic or equations (of course, no credit will be given for merely citing an answer).

Solution : Many students are surprised to find out that the two balls hit the ground with exactly the same speed. Think about the ball that is thrown upward : in the absence of friction, we expect the ball will rise to a certain height, then descend toward the Earth. What do you expect the speed of the ball to be at the instant it passes its launch point? If there is no friction, it should have the same speed as it did at launch.

In other words, the descending ball has a downward velocity of magnitude v when it is a height H above the ground. At this moment, its motion is identical to the ball originally thrown down; each has a downward velocity of v when they are H above the ground. Thus, they will have the same speed when they reach the ground.

We can also appeal to the equations of motion. Let's consider the equation:

$$v_f^2 = v_o^2 + 2\,a\,(x - x_o)$$

where v_f and v_o are final and initial speeds respectively, a is the acceleration, and $(x - x_o)$ is the total displacement. Since both objects have the same initial speed (and remember that any number squared is positive), the same acceleration (equal to the acceleration due to gravity), and both travel the same distance (from beginning to final points), we expect that the final speed in both cases is the same. We can determine the value of this speed from the equation. First, we have to define a coordinate system. Let’s choose the system in which down is positive. If down is positive, g has a positive value. The displacement (equal to final position - initial position) is H - 0 = H, and we have:

$$v_f^2 = v_o^2 + 2\,g\,H \Rightarrow v_f = \sqrt{v_o^2 + 2\,g\,H}$$

4. Consider the situation just described in problem 3. Determine the difference of the balls' arrival times on the ground. (Do not use numerical values for H or v; determine this using variables only.) Explain why this is the answer you could have predicted without doing any calculations.

Let' s start with the analysis of the last problem. The ball thrown initially upward will take some time to reach its apex and return to its launch point. Once it returns to its launch point, its motion to the ground will be the same as the ball originally thrown downward. Thus, the time differential in their landing times is just the time it takes an object launched vertically at a speed v to return to its launch level. We have shown in class that this time is simply 2 v/g. Let' s use the equations of motion to prove this.

The equation of motion for the ball tossed upward will be:

$$y_u = H + v\,t - \frac{1}{2}\,g\,t^2$$

The equation of motion for the ball initially thrown downward will be:

$$y_d(t) = H - v\,t - \frac{1}{2}\,g\,t^2$$

I am using a coordinate system in which up is positive; therefore the initial height is and gravity is negative. Note carefully the sign of the initial velocity of each ball. Since we want to know the time it takes for each ball to reach the ground (where y (t) = 0), we solve the two quadratic equations which yield :

$$\text{upward}: \; t_u = \frac{-v \pm \sqrt{v^2 - 4\,(-g/2)\,H}}{-g} = \frac{-v \pm \sqrt{v^2 + 2\,g\,H}}{-g} = \frac{v \pm \sqrt{v^2 + 2\,g\,H}}{g}$$

$$\text{downward}: \; t_d = \frac{v \pm \sqrt{v^2 - 4\,(-g/2)\,H}}{-g} = \frac{v \pm \sqrt{v^2 + 2\,g\,H}}{-g} = \frac{-v \pm \sqrt{v^2 + 2\,g\,H}}{g}$$

In the last step, we multiply through by (-1) to remove the negative in the denominator. Now, each computed time has two solutions; which do we choose? Look carefully at the discriminant; notice that the discriminant is the same in both cases. Since 2 g H is positive, we know that

$$v^2 + 2\,g\,H > v^2 \Rightarrow \sqrt{v^2 + 2\,g\,H} > v$$

In other words, when we compute the square root, the value will be greater than the value of v. Since our solution must yield a positive value for time, we know that we must choose in both cases the positive root. (Choosing the negative root for either time yields a negative result).

We subtract these two results to find the time difference in arrival; notice that the square roots are identical so will cancel out in the subtraction leaving:

$$\Delta t = t_u - t_d = \frac{v + \sqrt{v^2 + 2\,g\,H}}{g} - \frac{\left(-v + \sqrt{v^2 + 2\,g\,H}\right)}{g} = \frac{2\,v}{g}$$

as predicted above.

5. Problem 72, p. 63 of text. Five points for each part.

Solution : This problem will give us practice in using equations of motion, and will also allow us to think a bit more deeply about the meaning of “initial conditions”.

In this problem, we will make repeated use of the equation of motion in the vertical direction :

$$y(t) = y_o + v_{oy}\,t + \frac{1}{2}\,a\,t^2$$

We will use the same equation for parts a) and c) (no equations needed for b)), however, we will learn that we have to use different values for the initial conditions to describe different parts of the trip.

a) For the first part of the trip (the main engine burn or the acceleration phase), we set $y_o = 0$ and also $v_{oy} = 0$. We are told that the engine burns for 20s and the ascending rocket has a constant acceleration of $+2.5\, m/s^2$. After 20s, the height of the rocket is simply:

$$y(20\,s) = \frac{1}{2}\,a\,t^2 = \frac{1}{2}\left(2.5\,m/s^2\right)(20\,s)^2 = 500\,m$$

b) This part requires no calculations. At the highest point, we know the instantaneous velocity is zero. Since the rocket is in free fall, we also know that the only force acting on it is gravity, so that the acceleration of the rocket at the peak (and at all points after engine cut - off) is g (= -9.8 m/s^2).

c) This part requires us to compute the time of the total trip. Since we are asked to find time explicitly, we want to be able to use the equation of motion :

$$y(t) = y_o + v_{oy}\,t + \frac{1}{2}\,a\,t^2$$

But trip really consists of two different segments. We have already determined that the first segment (main engine burn) lasts 20s. But how long does the coasting phase of the trip last? We can use the equation of motion, but we have to determine what initial conditions we use for that portion of the trip.

We have already calculated that the coasting portion of the trip begins when the rocket has reached an altitude of 500m. Thus, we set y_o = 500m. Since the rocket is in free fall, we know its acceleration is equal to the acceleration due to gravity $\left(-9.8\, m / s^2\right)$. But we need to know the velocity of the rocket when it begins the coasting phase. We can determine this in two ways.

First method: The rocket started from rest, and then accelerated at a constant rate of $2.5\, m / s^2$ for 20s. We can use the basic definition of acceleration:

$$a = \frac{\Delta v}{\Delta t} \Rightarrow v_f - v_o = a\,t$$

Since the rocket started at rest, we simply have that the final velocity (for the acceleration phase) is simply $v = a\,t = 2.5\, m / s^2 \cdot 20s = 50m/s$. This will be the value of v_o for the coasting phase.

Second method: We can also use the relation:

$$v_f^2 = v_o^2 + 2\,a\,(y - y_o)$$

Where v_f is the final velocity (what we are looking for), v_o is the initial velocity (in this case = 0), a is the acceleration, y is the height achieved, and y_o is the initial height (set here to 0). These values give us:

$$v_f^2 = 2\left(2.5\, m / s^2\right)(500\, m) = 2500\, m^2 / s^2$$

$$v_f = 50\, m / s$$

as before.

Now that we have all of our values for the beginning of the coasting phase, we can write:

$$y(t) = y_o + v_{oy}\,t - \frac{1}{2}\,g\,t^2 = 500 + 50\,t - 4.9\,t^2$$

We want to find the time when the height of the rocket is zero, or:

$$500 + 50\,t - 4.9\,t^2 = 0 \Rightarrow t = \frac{-50\, m/s \pm \sqrt{(50\, m/s)^2 - 4\left(4.9\, m / s^2\right)(500\, m)}}{-9.8\, m / s^2}$$

or $t = 16.4\, s, \ -6.2\, s$

The meaningful solution for this portion of the trip is16.2s; the total time from launch to return is 16.4s+20s = 36.4s