PHYS 111
HOMEWORK #7

Due : 27 October 2016

All problems are taken from the text.

1. 18, p. 147

Solution : As discussed in class, this problem can be broken down into two parts. "During the jump" refers to the portion of the motion when the person' s feet are still in contact with the ground. During this phase, the ground must exert a normal force on the person that is larger than the weight of the person. The second part of the problem occurs when the person is in the air.

We are told that a person can jump to a maximum height of 60 cm (0.6 m) off the ground. This information allows us to determine the person' s speed at the moment of lift - off from the ground. We know that the person has an instantaneous velocity of 0 at the apex of motion, so we can use the equation :

$$v_f^2 = v_o^2 + 2\,a\,s$$

to determine the speed at lift-off. If we choose up as the positive direction, our acceleration is simply - g, and we can write:

$$0 = v_o^{\square} - 2\,g\,y_{max} \Rightarrow v_o^2 = 2\,g\,y_{max} \Rightarrow v_o = \sqrt{2\,g\,y_{max}}$$

For the values in this problem, we obtain the value of the launch velocity:

$$v_o = \sqrt{2 \cdot 9.8\,\mathrm{m}/\mathrm{s}^2 \cdot 0.6\,\mathrm{m}} = 3.43\,\mathrm{m/s}$$

We are then asked to find the force the ground must exert on the person to produce this launch speed. We will compute the acceleration of the person while in contact with the ground by applying the same equation to the time when the person is in contact with the ground. In this case, the initial velocity is zero and the final velocity for this phase is the launch velocity we just calculated. We are told that the person’s body moves through a distance of 0.5m while still in contact with the ground. This allows us to write:

$$v_f^2 = v_0^2 + 2\,a\,s \Rightarrow a = \frac{v_f^2 - v_0^2}{2\,s} = \frac{(3.43\,\mathrm{m/s})^2 - 0}{2\,(0.5\,\mathrm{m})} = 11.8\,\mathrm{m}/\mathrm{s}^2$$

Finally, we apply Newton' s second law to the person during the jump (while still in contact with the ground). The normal force from the ground acts up on the person, weight acts down, and these two forces combine to produce an acceleration :

$$\Sigma F_y = N - m g = m a \Rightarrow N = m(g+a) = m\left(9.8\,m/s^2 + 11.8\,m/s^2\right) = 21.6\,m.$$

(where m is the mass of the person). Since the person' s weight is m g or 9.8 m, we can see that the normal force is 21.6/9.8 times greater than the weight, or 2.2 W.

Now let' s do this problem entirely without numbers. Let’s call H the height of the jump, and s the distance the body moves while the feet are still in contact with the ground; let’s also call V_L the launch velocity, that is, the velocity of the person just as their feet lose contact with the ground.

Following the approach of the problem, we know that the launch velocity is related to the height of the jump by:

$$V_L^2 = 2 g H$$

Focusing now on the portion of the problem when the person is still on the ground, we know that the body starts at rest, accelerates from zero velocity to V_L while moving through a distance s. This allows us to write the acceleration of the body during that phase via:

$$V_L^2 = V_0^2 + 2 a s \Rightarrow a = \frac{V_L^2}{2 s} = \frac{2 g H}{2 s} = g\left(\frac{H}{s}\right)$$

Let' s look at this result for a second. The acceleration of the body depends on the ratio of H/s (the ratio of height achieved to distance through which the body moves). For instance, you would generate a very large acceleration if you could just flex your ankles (a small value of s) and jump very high (a large value of H).

Above we determined the normal force acting on the body. Using this expression for a (and the results from above) we can write :

$$N = m(g+a) = m\left(g + g\left(\frac{H}{s}\right)\right) = m g\left(1 + \frac{H}{s}\right)$$

And to express the normal force as a fraction of your weight, we have simply:

$$\frac{N}{m g} = \frac{m g\left(1 + \frac{H}{s}\right)}{m g} = 1 + \frac{H}{s}$$

And you can see that we obtain a very simple result and also that we never need to calculate the launch velocity to compute the normal force.

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2. 22, p. 147

Solution : Refer to the diagram on p. 147 of your text. If the car is accelerating, the string will hang at same angle from the vertical, described as β in the diagram. The tension in the string will have both a horizontal and vertical component. The mass is not accelerating in the y direction, so we know the vertical component of tension must equal the weight, or :

$$\Sigma F_y = T\cos\beta - m\,g = 0 \text{ or } T\cos\beta = m\,g$$

The mass is accelerating in the x direction, and the horizontal component of tension is the force causing the string to accelerate:

$$\Sigma F_x = T\sin\beta = m\,a$$

To find an expression for the angle β produced by an acceleration a, we divide the two equations :

$$\frac{T\sin\beta}{T\cos\beta} = \frac{m\,a}{m\,g} \Rightarrow \tan\beta = \frac{a}{g}$$

or :

$$\beta = \tan^{-1}\left(\frac{a}{g}\right)$$

This allows us to find the magnitude of β for any acceleration. When the magnitude of a is 3 m/s^2, β is:

$$\beta = \tan^{-1}\left(\frac{3\,m/s^2}{9.8\,m/s^2}\right) = 17^o$$

Which way does the string hang? Remember that Newton' s second law is a vector equation, so that the acceleration is in the direction of the net force. If the object is accelerating to the right, the horizontal force must act to the right. Figure 5.50 shows this situation; the acceleration is to the right, and so the string must deflect to the left, so that the horizontal component of motion in the string is to the right. In the second case, the object moves to the left with a negative acceleration. That means the the velocity vector points to the left, but the acceleration vector points to the right, and string hangs in the same direction as depicted in Fig. 5.50. The value of β here is simply :

$$\beta = \tan^{-1}\left(\frac{4.5\,m/s^2}{9.8\,m/s^2}\right) = 24.7^o$$

Finally, we are asked what happens when the object moves at a constant speed. The value of the constant speed does not matter, it only matters that the speed is constant and the acceleration is zero. If the acceleration in the x direction is zero, there is no net force in the x direction, and the string hangs vertically.

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3. 28, p. 148

Solution : The key phrase in this problem is that the system moves to the right at a constant speed. This tells you there is no acceleration and thus the sum of forces in each direction is zero. The value of the constant is speed does not matter; the fact that there is no acceleration is the key point.

First let's focus on box A. There are both horizontal and vertical forces acting on A. In the horizontal direction, there is the tension in the string (acting to the right), and friction with the ground opposing motion. In the vertical direction, there is the normal force and weight. Applying Newton's second law we get:

$$\Sigma F_x = T - f_k = 0 \Rightarrow T = f_k \tag{1}$$

Where T is the tension in the string and f_k is the force of kinetic friction acting on the box. In the vertical direction, we have:

$$\Sigma F_y = N - m_A g = 0 \Rightarrow N = m_A g$$

As expected, the normal force equals the weight of box A. We use this expression for the normal force to rewrite the frictional force :

$$f_k = \mu_k N = \mu_k m_A g$$

Using this expression for frictional force in equation (1) gives us for the tension in the string:

$$T = f_k = \mu_k m_A g$$

Now, let' s analyze the forces on box B. In the horizontal direction, there are the pulling force F (to the right), friction opposing the motion, and the tension in the string acting to the left. In the vertical direction, weight and the normal force balance. We can write :

$$\Sigma F_x = F - f_k - T = 0 \Rightarrow F = T + f_k$$

$$\Sigma F_y = N - m_B g = 0 \Rightarrow N = m_B g$$

Since the friction force = $\mu_k N = \mu_k m_B$ g, we have:

$$F = T + f_k = \mu_k m_A g + \mu_k m_B g = \mu_k (m_A + m_B) g$$

The last result makes use of our expression for T determined in the first part. Notice that the pulling force F is the same result we would get if we had a single mass equal to the sum of masses of A and B. This makes sense; the pulling force doesn't "know" there is a connecting string, only that there is a total frictional force that must be equalled in order to produce motion at a constant speed.

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4. 34, p. 148

Solution : In the first part, we are asked to find the angle of the ramp which will initiate motion. Just before the box starts to slide, the forces on the box are the component of gravity down the plane (m g sin θ), the component of gravity perpendicular to the plane (m g cos θ), , the normal force, and static friction which acts along the plane. Newton' s second law applied to the box is :

$$\Sigma F_\perp = N - m g \cos\theta_1 = 0 \Rightarrow N = m g \cos\theta_1$$

$$\Sigma F_{||} = m g \sin\theta_1 - f_S = 0$$

We use static friction because we want to find the force necessary to overcome static friction to get the box moving. The force of static friction is described by:

$$f_s = \mu_s N = \mu_s m g \cos\theta_1$$

Combining these results gives us:

$$\Sigma F_{||} = m\,g \sin\theta_1 - \mu_s\, m\,g \cos\theta_1 = 0 \Rightarrow m\,g\,(\sin\theta_1 - \mu_s \cos\theta_1) = 0$$

This means that $\sin\theta_1 = \mu_s \cos\theta_1 \Rightarrow \tan\theta_1 = \mu_s$

In the second case, the box slides at a constant speed down a ramp of angle θ_2. Since the motion is at constant speed along a straight line, the acceleration is zero, and our force balance becomes:

$$m\,g \sin\theta_2 - \mu_k \cos\theta_2 = 0 \Rightarrow \tan\theta_2 = \mu_k$$

So we can use this technique to measure the coefficients of kinetic and static friction.

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5. 40, p. 149. First do this problem with symbols rather than numbers. The box has mass M, the pushing force has magnitude P acting at an angle θ below the horizontal. The coefficient of friction is μ. Answer all parts a) through e) using these symbols. Then use the numbers given in the book and compute the numerical answers for all parts a) through e). (5 pts each part).

Solution : I will do parts a)-d) first, and do part e) at the end. The forces acting on the box are weight, the normal force, the components of P (which act down and to the left) and the force of static friction. Choosing up and to the left as positive, Newton' s second law gives :

$$\Sigma F_y = N - m\,g - P \sin\theta = 0 \Rightarrow N = m\,g + P \sin\theta$$

(we will need this expression for N to determine the friction force), and in the horizontal direction:

$$\Sigma F_x = P \cos\theta - f_s = 0 \Rightarrow f_s = P \cos\theta$$

Since $f_s = \mu_s N$ we can write $f_s = \mu_s (m\,g + P \sin\theta)$.

To determine the largest force P that could act on the box without causing it to move, we equate the static friction force to the horizontal component of the pushing force:

$$P \cos\theta = \mu_s (m\,g + P \sin\theta)$$

and after a little algebra, we can write:

$$P_{max} = \frac{\mu_s\, m\,g}{\cos\theta - \mu_s \sin\theta}$$

This is the greatest P that you can exert on the box before motion ensues, so the maximum value of the static friction force is simply $P_{max} \cos\theta$. This last expression has the interesting property that when the denominator is zero, the force necessary to overcome friction becomes infinite (in other words, if you push down at a critical angle, there is no force that will allow you to move the box horizontally). This critical angle occurs when $\cos\theta = \mu_s \sin\theta$ or when $\tan\theta = 1/\mu_s$.

Now, let' s try this with numbers. The normal force on the box is :

$$N = m\,g + P \sin\theta = 125\,N + 75 \sin 30\,N = 162.5\,N$$

The frictional force for these values of P and θ is:

$$f_s = P \cos\theta = 75\,N \cos 30 = 65\,N$$

This must be less than the maximum frictional force; this is given by:

$$f_s = \mu_s N = 0.8 \cdot 162.5\,N = 130\,N$$

Another way to interpret maximum force is is to ask what is the greatest force P that can act on the box (at this angle) without causing motion is:

$$P_{max} = \frac{\mu_s m g}{\cos\theta - \mu_s \sin\theta} = \frac{0.8 \cdot 125\,N}{\cos 30 - 0.8 \sin 30} = 214\,N$$

Since the static friction force equals the horizontal component of this pushing force, the maximum possible value of static friction is 0.8·214N = 172N

If the force is now pulling upward at an angle of θ, our second law equations become:

$$\Sigma F_x = P\cos\theta - f_s = 0 \Rightarrow P\cos\theta = f_s$$

$$\Sigma F_y = N - W + P\sin\theta = 0 \Rightarrow N = W - P\sin\theta$$

This last result shows that if the vertical component of P equals the weight, there is no normal force between the box and the floor; this is equivalent to saying that the box loses contact with the floor, since the vertical force is now sufficient to lift the box against gravity. Using the values of the numbers we are given:

$$N = W - P\sin\theta = 125\,N - 75\sin 30 = 87.5\,N$$

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6. 46, p. 149

Solution : Consider the mass suspended from a spring; if the mass is not accelerating, then the net force acting on it is zero. The only forces acting on the spring are the elastic force of the spring acting up, and weight acting down. This means :

$$k(\Delta y) - mg = 0 \Rightarrow k = \frac{m g}{\Delta y}$$

where k is the spring constant and Δy is the amount the spring is stretched. For the values we are given, Δy = 14.40cm - 12.00 cm = 2.40 cm = 0.024 m, and:

$$k = \frac{0.875\,kg \cdot 9.8\,m/s^2}{0.024\,m} = 357\,N/m$$

In order to make the total length 17.72 cm, we need to stretch the spring by an amount 5.72 cm or 0.0572 m, this will require a mass of:

$$k(\Delta y) = m g \Rightarrow m = \frac{k(\Delta y)}{g} = \frac{357\,N/m \cdot 0.0572\,m}{9.8\,m/s^2} = 2.08\,kg$$

This result makes sense; if we want to increase the extension by a ratio of (5.72/2.40 = 2.38), we expect to need a mass that is 2.38 greater than the original mass.

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7. 48, p. 149

Solution : Remember that Newton' s second law deals with the forces acting on a single object. If we focus our attention on the 3 kg mass, the only force acting on it is the elastic force due to the compression of the spring. As far as the 3 kg mass is concerned, it does not matter if the spring is attached to another mass, a wall, or anything else. The only force acting on the spring is elastic force, and the amount of the elastic force is determined by the stiffness and compression of the spring. Therefore, for both masses, we use 0.05 m for the compression (Δx) of the spring. Newton' s second law gives us :

$$F = k(\Delta x) = m a \Rightarrow a = \frac{k(\Delta x)}{m}$$

For each mass we obtain:

$$a_{3\,kg} = \frac{500\,N/m\,(0.05\,m)}{3\,kg} = 8.33\,m\big/s^2 \qquad a_{10}\,kg = \frac{500\,N/m\,(0.05\,m)}{10\,kg} = 2.5\,m\big/s^2$$

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8. 56, p. 150

Solution : In this problem, we get to consider both the second and third laws. Let' s start by consider the sphere on the left. The forces acting on it are the tension in the rope (which has both horizontal and vertical components), weight, and the normal force due to contact with the other sphere (which acts horizontally). Let me call these forces T_L, W_L, and N_L (in other words, the subscript L refers to forces acting on the left sphere). Similarly for the right sphere the forces are T_R, W_R, and N_R. Since the spheres are of equal radius, the lengths of the wires are equal, and the angle each wire makes with the vertical is 25^o. Now, let's apply Newton's Laws:

Since the system is in equilibrium, all forces sum to zero in both the x and y directions. For the left sphere, we can write:

$$\Sigma F_x = T_L \sin\theta - N_L = 0 \Rightarrow T_L \sin\theta = N_L$$

This means that the horizontal component of the tension is equal in magnitude and opposite in direction to the normal force of contact acting on the left sphere. I am using $\theta = 25^o$. In the vertical direction, we have:

$$\Sigma F_y = T_L \cos\theta - W = 0 \Rightarrow T_L \cos\theta = W \text{ or } T_L = \frac{W}{\cos\theta} = \frac{71.2\,N}{\cos 25} = 79\,N$$

This allows us to calculate the contact force on the left sphere:

$$N_L = T_L \sin\theta = 79\,N \cdot \sin 25 = 33\,N$$

According to the third law, the contact force acting on the left sphere must be equal and opposite to the contact force acting on the righ sphere, so we can conclude the same force acts on the right sphere.

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9. 57, p. 150

Solution : We are told the plane if frictionless, that means the only forces acting along the plane are the elastic forces due to the compressed spring (acting up the plane) and the component of gravity acting down the plane. We are asked to find the acceleration of the mass at various points.

As is now commonplace, we begin by writing Newton' s second law for the mass. Setting up the plane as the positive direction, summing forces along the plane yields :

$$\Sigma F_{||} = k(\Delta x) - m g \sin\theta = m a$$

And the acceleration of the mass is:

$$a = \frac{k(\Delta x) - m g \sin\theta}{m}$$

We can find all three acceleratons from this expression. When the compression is 0.1m, we have:

$$a = \frac{1000\ \mathrm{N/m}\ (0.1\ \mathrm{m}) - 2\ \mathrm{kg}\left(9.8\ \mathrm{m}/\mathrm{s}^2\right)\sin 30}{2\ \mathrm{kg}} = 45.1\ \mathrm{m}/\mathrm{s}^2$$

Since we set up the plane as positive, this answer means the acceleration is directed up the plane. When the compression is 0.05m:

$$a = \frac{1000\ \mathrm{N/m}\ (0.05\ \mathrm{m}) - 2\ \mathrm{kg}\left(9.8\ \mathrm{m}/\mathrm{s}^2\right)\sin 30}{2\ \mathrm{kg}} = 20.1\ \mathrm{m}/\mathrm{s}^2$$

also up the plane.

Finally, if the spring is fully extended, Δx is zero, and the acceleration is simply - g sin θ or - 4.9 m/s^2 (directed down the plane).