PHYS 111
HOMEWORK #9

Due : 10 November 2016

1. A geosynchronous satellite orbits the Earth in a period of 24 hours, thereby remaining over the same spot on the surface of the Earth. Determine the radius of this orbit (measured from the center of the Earth).

Solution : For this problem we make use of Kepler' s third law (derived in class) :

$$\mathrm{M\,P}^2 = \frac{4\,\pi^2}{\mathrm{G}}\,\mathrm{a}^3$$

where M is the mass of the central object (in this case the Earth), P is the period of orbit (1 day = 86400s), G is the Newtonian Gravitational constant, and a is the radius of the orbit (actually, the semi-major axis but if we assume circular orbits, this is the same as the radius). Solving for the radius of the orbit gives us:

$$\mathrm{a} = \left[\frac{\mathrm{G\,M\,P}^2}{4\,\pi^2}\right]^{1/3} = \left[\left(6.67\times10^{-11}\ \mathrm{m^3\,kg^{-1}\,s^{-2}}\right)\left(6\times10^{24}\ \mathrm{kg}\right)(86\,400\ \mathrm{s})^2 \big/ \left(4\,\pi^2\right)\right]^{1/3}$$

$$= 4.2\times10^7\ \mathrm{m} = 42,\,000\ \mathrm{km} \approx 26,\,000\ \text{miles from the center of the Earth}$$

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2. Determine the period of a satellite in low Earth orbit (about 200 km above the surface of the Earth).

Solution : We start with the same equation and solve for period :

$$\mathrm{P} = \sqrt{\frac{4\,\pi^2\,\mathrm{a}^3}{\mathrm{G\,M}}} = \sqrt{\frac{4\,\pi^2\left(6.4\times10^6\ \mathrm{m}\right)^3}{\left(6.67\times10^{-11}\ \mathrm{m^3\,kg^{-1}\,s^{-2}}\right)\left(6\times10^{24}\ \mathrm{kg}\right)}} = 5082\ \mathrm{s} = 1.41\ \text{hours} = 85\ \text{mins}$$

And this is the approximate orbital period of a satellite in low Earth orbit.

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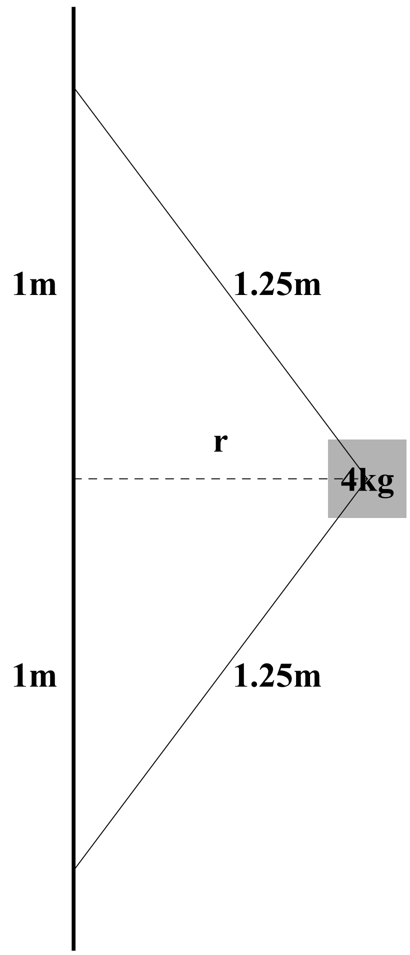
3. Problem 51, p. 178.

Solution :

Let' s refer to the diagram below :



We will begin our analysis by considering the forces acting on the 4 kg block Since the block is not accelerating in the y direction, we know the sum of forces in the y direction is zero, or :

$$\Sigma F_y = T_{u,y} - T_{l,y} - m\,g = 0$$

where $T_{u,y}$ is the y component of the tension in the upper string and $T_{u.l}$ is the y component of the tension in the lower string. We can write this as:

$$\Sigma F_y = T_u \cos\theta - T_l \cos\theta - m\,g = 0$$

where we define θ to be the angle between the vertical rod and the string. We can use the data provided in the diagram to see that $\cos\theta = 1/1.25 \Rightarrow \theta = 37^o$. Knowing that the tension in the upper string is 80N, we can solve for the tension in the lower string:

$$(T_u - T_l)\cos\theta = m\,g \Rightarrow T_l = T_u - \frac{m\,g}{\cos\theta} = 80\,N - \frac{4\,kg \cdot 9.8\,m/s^2}{0.8} = 31\,N$$

We are then asked to find the speed of the block in its rotation. We find this by considering the horizontal forces. The horizontal force is generated by the horizontal component of tension in the two strings, and this horizontal force generates the centripetal force:

$$\Sigma F_x = T_{u,x} + T_{l,x} = \frac{m v^2}{r}$$

The horizontal components of tension are:

$$T_u \sin\theta + T_l \sin\theta = \frac{m v^2}{r} \Rightarrow v = \sqrt{\frac{r \sin\theta\,(T_u + T_l)}{m}} =$$

Before we can compute a speed, we need to determine the radius of the orbit. We can find this from the Pythoagorean theorem:

$$r = \sqrt{(1.25\text{ m})^2 - (1\text{ m})^2} = 0.75\text{ m}$$

Putting all this together, we get:

$$v = \sqrt{0.75\text{ m} \sin 37^{\circ}\,(80\text{ N} + 31\text{ N}) / 4\text{ kg}} = 3.54\text{ m/s}$$

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4. A mass starts at rest from position A on the sphere as shown below. The mass is set in motion and slides without friction to point B.

a) What is the vertical difference between points A and B?

Solution : We first compute the vertical drop from A to B so we can use the conservation of energy to compute the speed at B. The distance from the center of the circle to A is the radius, R, of the circle. The "height" of B above the center is R cos θ, so the vertical drop from A to B is R - R cos θ = R (1 - cos θ)

b) What is the speed of the mass at B (assuming it started at rest at A)? Express your answer in terms of R, g and θ.

Solution : We make use of the conservation of energy. In the absence of friction, we know that the total energy at A equals the total energy at B. The total energy is the sum of kinetic and potential, so we can write :

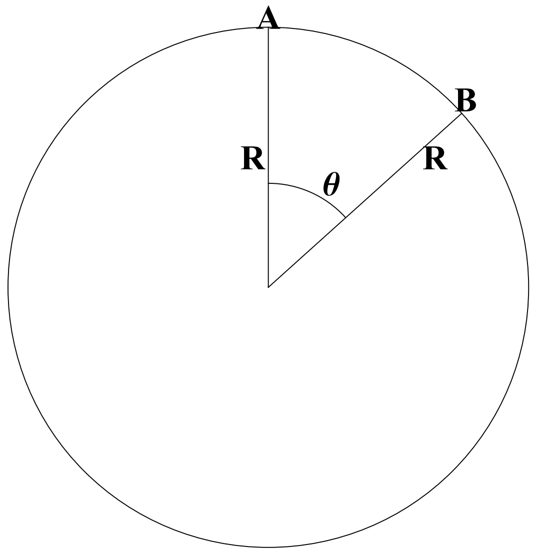
$$K_A + U_A = K_B + U_B$$

At A, K = 0 and U = m g R; at B K = 1/2 m v^2 and U = m g R cos θ. Thus we have:

$$m g R = m g R \cos\theta + \frac{1}{2} m v^2 \Rightarrow m g R (1 - \cos\theta) = \frac{1}{2} m v^2$$

Another way to interpret this equation is that the loss in potential energy of m g R (1-cos θ) equals the gain in kinetic energy. This yields for the speed at B: :

$$v = \sqrt{2 g R (1 - \cos\theta)}$$



5. Problem 4, p. 213

Solution : We will use the equation

$$W = F\cos\theta\, s$$

for the various parts of this problem. In the first case, we are told a constant force of 8.5N drags a box through a displacement of 17.4m. The work done is quite simply

$$W = 8.5\,\mathrm{N} \cdot 17.4\,\mathrm{m} = 148\,\mathrm{J}$$

In the second case, we are told the force is directed at angle with respect to the floor, and does a total of 65J of work. We find the angle between the force and displacement via:

$$W = 65\,\mathrm{J} = 8.5\,\mathrm{N}\,(17.4\,\mathrm{m})\cos\theta \Rightarrow \theta = \cos^{-1}\left(\frac{65\,\mathrm{J}}{(8.5 \cdot 17.4\,\mathrm{J})}\right) = 64^{\mathrm{o}}$$

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6. Problem 8, p. 214. Answer parts a) - d) and also : If the package is at rest at the top of the chute, what is its speed at the bottom? (Five points each part).

Solution : The box travels 2 m down the ramp; the forces acting on the box are gravity and friction. The component of gravity down the plane is m g sin θ and the component of gravity perpendicular to the plane is m g cos θ. Friction acts along the plane with a force equal to μ_k m g cos θ. This allows us to compute:

a) Work done by gravity $= (\mathrm{m\,g}\sin\theta)\,\mathrm{s} = 8\,\mathrm{kg}\left(9.8\,\mathrm{m}/\mathrm{s}^2\right)\sin 53\,(2\,\mathrm{m}) = 125\,\mathrm{J}$

b) work done by friction $= (\mu_k\,\mathrm{m\,g}\cos\theta)\,\mathrm{s} = 0.4 \cdot 8\,\mathrm{kg} \cdot 9.8\,\mathrm{m}/\mathrm{s}^2 \cdot \cos 53 \times 2\,\mathrm{m} = 37.8\,\mathrm{J}$

c) The normal force does no work since it is perpendicular to the displacement.

d) The total work is 125 J - 37.8 J = 87.2 J

e) Let' s solve this problem three different ways. First, we can use either the work - energy theorem to find the speed at the bottom. We know that the work done on the mass will equal the change in kinetic energy. Since the initial kinetic energy is zero, the work done will equal the final kinetic energy, so we have :

$$\text{Work done} = 87.2\text{ J} = \frac{1}{2} m v^2 \Rightarrow v = \sqrt{2(87.2 J)/8\text{ kg}} = 4.67\text{ m/s}$$

Second, we can obtain the same answer by using methods of energy conservation:

$$K_i + U_i + K_f + U_f + W_{other}$$

where the subscripts i, f refer respectively to initial and final states, and W_{other} is work done by dissipative forces. For the situation presented here, we have:

$$0 + m g h = \frac{1}{2} m v^2 + W_{other}$$

At the top of the ramp, the box is 2 sin 53 = 1.6m above the ground, and we know the work done by friction is 37.8J, this gives us:

$$8\text{ kg} \cdot 9.8\text{ m/s}^2 \cdot 1.6\text{ m} = \frac{1}{2} 8\text{ kg}\, v^2 + 37.8\text{ J} \Rightarrow v = 4.68\text{ m/s}$$

Finally, we can solve this via kinematic methods. We can Use

$$v_f^2 = v_0^2 + 2 a s$$

to compute the final speed once we compute the acceleration. We find the acceleration from applying Newton’s second law to forces acting along the plane. Considering these forces we get:

$$\Sigma F_{||} = m g \sin\theta - \mu_k m g \cos\theta = m a \Rightarrow a = g(\sin\theta - \mu_k \cos\theta) = 5.47\text{ m/s}^2$$

Then, we get :

$$v_f^2 = 0 + 2\left(5.47\text{ m/s}^2\right)(2\text{ m}) \Rightarrow v_f = 4.68\text{ m/s}$$

which gives the same result for finding the speed at the bottom of the ramp.

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7. Problem 18, p. 214.

Solution : The statement of the problem (in part b) suggests we might want to try to do som comparative reasoning, so let' s try to do everything in symbols, using numbers only at the end. A skier of mass m has an initial speed of v and then coasts for a distance d on level ground. We are asked to use the work energy theorem to find the coefficient of kinetic friction.

The work energy theorem relates the amount of work done to the change in kinetic energy. In this case we have :

$$\Delta KE = KE_f - KE_i = 0 - \frac{1}{2} m v^2$$

The force doing the work is friction; the force of friction between the skier and the snow is μ_k m g, so the total work done is:

$$W_f = \mu_k \, m \, g \, d \cos 180 = -\mu_k \, m \, g \, d$$

(the factor of cos 180 arises because the force acts in the direction opposite displacement). Equating these expressions gives us :

$$\frac{1}{2} m v^2 = \mu_k \, m \, g \, d \Rightarrow \mu_k = \frac{v^2}{2 \, g \, d} = \frac{(12 \, m/s)^2}{2 \cdot 9.8 \, m/s^2 \cdot 184 \, m} = 0.04$$

Now, having determined an expression for coefficient of friction as a function of initial speed and stopping distance, we can see that the mass of the skier is irrelevant and plays no role in determining the coefficient of friction. If the skier' s initial speed is doubled, and the coasting distance remains the same, we can see from our expression that μ_k varies as the square of the initial speed. If the initial speed is squared, μ_k is quadrupled, and in this case would equal 0.16.

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8. Problem 37, p. 215

Solution : Conservation of energy allows us to relate the final kinetic energy to the initial potential energy of the hailstone :

$$K_{aloft} + U_{aloft} = K_{ground} + U_{ground}$$

Using the ground as our reference level for potential energy, U_{aloft} = m g h where h is 500m, The potential on the ground is then zero, as is the kinetic at the start aloft. This gives us:

$$0 + m \, g \, h = \frac{1}{2} m v^2 + 0 \Rightarrow v = \sqrt{2 \, g \, h} = \sqrt{2 \cdot 9.8 \, m/s^2 \cdot 500 \, m} = 99 \, m/s = 220 \, mi/hr.$$

Hailstones at this speed would fracture our skulls; fortunately, most the initial potential energy is converted to work done against the atmosphere rather than to final kinetic energy.

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9. Problem 44, p. 216.

Solution : This is a conservation of energy problem, in which the initial elastic potential energy is converted into either kinetic or elastic potential energy. Originally, the mass m compresses a spring of constant $k_1$by an amount (Δx_1). This means the spring has elastic potential energy of

$$U_{el1} = \frac{1}{2} k_1 (\Delta x_1)^2$$

If the spring then travels without losing any energy, it will compress the second spring (of constant k_2 by an amount (Δx_2), meaning the second spring acquires elastic potential energy equal to

$$U_{el2} = \frac{1}{2} k_2 (\Delta x_2)^2$$

Since there is no loss of energy, the initial potential energy must equal the final potential energy, or

$$\frac{1}{2} k_1 (\Delta x_1)^2 = \frac{1}{2} k_2 (\Delta x_2)^2$$

The first part of the problem asks us to find the compression of the second spring:

$$\Delta x_2 = \Delta x_1 \sqrt{\frac{k_1}{k_2}} = 0.04\ \mathrm{m} \sqrt{\frac{32\ \mathrm{N/cm}}{16\ \mathrm{N/cm}}} = 0.04\ \mathrm{m}\sqrt{2} = 0.057\ \mathrm{m}$$

Once the mass has left contact with the first spring, we know all of the initial elastic potential energy was converted to kinetic energy, or

$$\frac{1}{2} k_1 (\Delta x_1)^2 = \frac{1}{2} m v^2 \Rightarrow v = \Delta x_1 \sqrt{\frac{k_1}{m}} = 0.04\ \mathrm{m} \sqrt{\frac{3200\ \mathrm{N/m}}{1.5\ \mathrm{kg}}} = 1.8\ \mathrm{m/s}$$

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10. Problem 52, p. 216. Use the diagram provided in the text for reference, but solve the problems using only symbols. In other words, the mass is m, the spring of constant k is compressed by an amount Δx, it travels for a distance L. Use this information to determine an expression for the coefficient of friction between the block and the table.

Solution : A mass m compress a spring of constant k by an amount Δx. When released, the mass moves a distance d before coming to rest. We are asked to find the coefficient of friction μ_k. In this case, we know all of the initial elastic potential energy goes into doing work against friction. So we can write:

$$U_{el} = W_f$$

or

$$\frac{1}{2} k (\Delta x)^2 = f_k L$$

Now, since the force of friction is given by :

$$f_k = \mu_k N = \mu_k m g$$

we have :

$$\frac{1}{2} k (\Delta x)^2 = \mu_k m g L \Rightarrow \mu_k = \frac{k (\Delta x)^2}{2 m g L}$$