AN INTRODUCTION TO MATRIX OPERATIONS IN MATHEMATICA

We can use Mathematica to save a lot of computational time and potential error in a variety of matrix operations. Let' s consider some of the most important ones we will encounter.

First, we input a matrix as a list of vectors, so that we input :

In[639]:=
```
Clear[matrix]
matrix = {{1, 2, 3}, {3, 1, 2}, {3, -2, 4}};
```

We obtain :

In[641]:=
```
matrix // TraditionalForm
```

Out[641]//TraditionalForm=

$$\begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \\ 3 & -2 & 4 \end{pmatrix}$$

Now, suppose this matrix represents the system of equations :

$$x + 2y + 3z = 4$$
$$3x + y + 2z = 7$$
$$3x - 2y + 4z = 5$$

We can construct the matrix equation :

$$\begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \\ 3 & -2 & 4 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ 7 \\ 5 \end{pmatrix}$$

We can solve for the values of x, y, and z that satisfy this system of equations two different ways; one is to use the Solve function :

In[645]:=
```
Solve[
  {x + 2 y + 3 z == 4, 3 x + y + 2 z == 7, 3 x - 2 y + 4 z == 5}, {x, y, z}]
```

Out[645]= $\left\{\left\{x \to \frac{61}{31},\ y \to \frac{24}{31},\ z \to \frac{5}{31}\right\}\right\}$

Alternately, we can use matrix techniques; as we learned in class, the matrix equation :

$$M\,\mathbf{u} = \mathbf{v}$$

can be solved :

$$M^{-1} M\,\mathbf{u} = M^{-1}\,\mathbf{v} \Rightarrow I\,\mathbf{u} = \mathbf{u} = M^{-1}\,\mathbf{v}$$

If we can find M inverse and then the product of $M^{-1}\mathbf{v}$, we can solve for **u.**

In[646]:=
```
Inverse[matrix].{4, 7, 5}
```

Out[646]= $\left\{\frac{61}{31}, \frac{24}{31}, \frac{5}{31}\right\}$

Produces the same results. The Mathematica way to find the inverse of a matrix is simply using the Inverse function once you have defined a matrix.

Determinants

We have also discussed in class that we know an inverse of a matrix exists if its determinant is non - zero; we already know an inverse exists for our original matrix, which we verify with the Det command :

In[647]:=
```
Det[matrix]
```

Out[647]= -31

Do you see any relationship between the value of the determinant and the solutions to the system of equations?

Use matrix methods to solve for x, y, and z in the following system of equations :

$$x + y + 2z = 1$$
$$2x - 2y - z = 3$$
$$3x - y + z = 4$$

What solutions do you get? What issues are you encountering? Can you look at these equations and figure out why this might be the case?

Transpose of a matrix :

We learned in class that the transpose of a matrix is obtained by interchanging rows and columns; the Mathematica command is :

In[651]:=
```
matrixᵀ
```

Out[651]=
```
{{1, 2, 3}, {1, -2, -1}, {2, -1, 1}}
```

where you produce the superscript "T" by entering :

```
ESC tr ESC
```

Now, let' s see if the rotation matrix is orthogonal. We know that the rotation matrix is :

In[664]:=
```
Clear[rotmatrix, x]
rotmatrix = {{Cos[x], -Sin[x]}, {Sin[x], Cos[x]}};
rotmatrix.rotmatrixᵀ // TraditionalForm
```

Out[666]//TraditionalForm=

$$\begin{pmatrix} \cos^2(x)+\sin^2(x) & 0 \\ 0 & \cos^2(x)+\sin^2(x) \end{pmatrix}$$

The last operation multiplies the rotation matrix by its transpose, and produces the identity matrix. This means that the transpose of the rotation matrix is its inverse, and this satisties the definition of an orthogonal matrix :

$$A^{-1} = A^{T}$$