USEFUL MATHEMATICA FUNCTIONS AND TECHNIQUES

We are at a point in the semester when you have learned the basics of Mathematica and its uses in scientific programming. Now, let me introduce you to several useful features of Mathematica. I will show you the basics of each item, you can delve into more depth on your own using the online documentation center.

I. Solving equations :

The Solve feature is useful for solving one or more polynomial equations :

In[117]:=
```
Solve[x^2 - 3 x + 4 == 0, x]
```

Out[117]= $\left\{\left\{x \to \frac{1}{2}\left(3 - i\sqrt{7}\right)\right\}, \left\{x \to \frac{1}{2}\left(3 + i\sqrt{7}\right)\right\}\right\}$

In[120]:=
```
Clear[m, M, g, a]
Solve[{T - m g == m a, T - M g == - M a}, {T, a}]
```

Out[121]= $\left\{\left\{T \to \frac{2\,g\,m\,M}{m + M},\ a \to -\frac{g\,(m - M)}{m + M}\right\}\right\}$

If you want to solve equations with trig or exponential function, use FindRoot :

In[123]:=
```
FindRoot[Exp[x] == 3 x, {x, 1}]
```

Out[123]=
```
{x → 0.619061}
```

This finds the solution to the equation :

$$e^x = 3x$$

in the vicinity of x = 1. Plotting the functions ahead of time is useful to determine where you should begin testing for a solution.

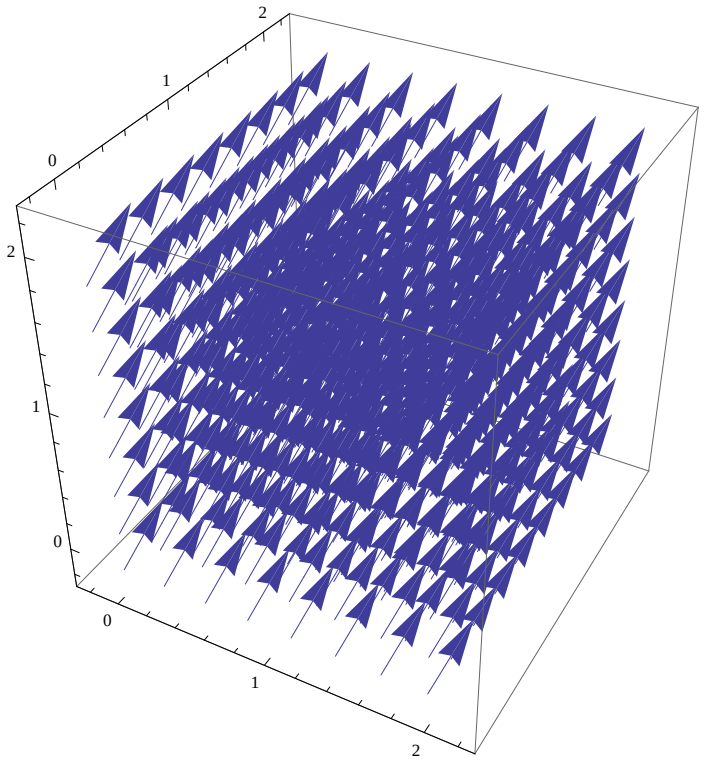
II. Vectors and Matrices

You can write a vector in Mathematica using braces :

```
Clear[v]
v = {1, 2, 3}
VectorPlot3D[v, {x, 0, 1}, {y, 0, 1}, {z, 0, 1}]
```

Out[128]= {1, 2, 3}

Out[129]=



This creates the vector :

$$\mathbf{v} = 1\,\hat{\mathbf{x}} + 2\,\hat{\mathbf{y}} + 3\,\hat{\mathbf{z}}$$

and plots it in the indicated region.

We can write a 3 x 3 matrix as an array of arrays :

In[130]:=
```
Clear[matrix]
matrixA = {{1, 2, 3}, {-1, 0, 1}, {2, -3, 4}};
matrixA // MatrixForm
```

Out[132]//MatrixForm=

$$\begin{pmatrix} 1 & 2 & 3 \\ -1 & 0 & 1 \\ 2 & -3 & 4 \end{pmatrix}$$

To find the inverse of a matrix :

In[136]:=
```
Inverse[matrixA]
```

Out[136]= $\left\{\left\{\frac{1}{8}, -\frac{17}{24}, \frac{1}{12}\right\}, \left\{\frac{1}{4}, -\frac{1}{12}, -\frac{1}{6}\right\}, \left\{\frac{1}{8}, \frac{7}{24}, \frac{1}{12}\right\}\right\}$

and verifying that the matrix times its inverse is the identity matrix :

In[138]:=
```
matrixA.Inverse[matrixA]
```

Out[138]= {{1, 0, 0}, {0, 1, 0}, {0, 0, 1}}

You just use a standard period to indicate matrix multiplication.

III. "/."

The "slash dot" command is very useful in computing values in Mathematica. The expression :

```
In[139]:= Sin[n π / 4] /. n → {1, 2, 3, 4, 5, 6, 7, 8}
```

$$\text{Out[139]= } \left\{\frac{1}{\sqrt{2}}, 1, \frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}}, -1, -\frac{1}{\sqrt{2}}, 0\right\}$$

replaces n in Sin[n π/4] by the values in the braces.