PHYS 301
HOMEWORK #1

Due : 16 Jan. 2015

All assignments must be turned in at the beginning of class on the day they are due. No credit will be given to homework submitted after the solutions are posted. All answers must be accompanied by complete and clear work. Refer to the syllabus for the proper format for submitting homework.

You may use Mathematica to check your results, but your solutions must show all steps explicitly.

1. If m and n are positive integers, find the value of the integrals :

$$\int_{-\pi}^{\pi} \cos(n\,x)\ dx \qquad \int_{-\pi}^{\pi} \sin(m\,x)\,dx$$

2. Use the sin and cos addition formulae to show that

$$\cos(n+m)\,x\ +\cos(n-m)\,x\ =\ 2\cos(n\,x)\cos(m\,x)$$

3. Use the result of problem 2 to determine the value of

$$\int_{-\pi}^{\pi} \cos(n\,x)\cos(m\,x)\,dx$$

where n and m are integers. Make sure you consider separately the cases $m = n$ and $m \neq n$

4. The cost of producing two products, x and y, is given by :

$$C(x,\,y)\ =\ 3\,x^2 + 2\,x\,y\ + 2\,y^2$$

If 10 units of x and 5 units of y are being produced, and the rate of production of x is increasing by 2 units/day and the rate of production of y is decreasing by 1 unit/day, what is the instantaneous rate of change of $C(x,\,y)$?

5. The temperature distribution in a room is given by :

$$T(x,\,y,\,z)\ =\ z^2 + e^{\left(x^2 - y^2\right)}$$

where x, y and z are the familiar Cartesian coordinates. A particle moves through the room with a velocity vector given by $v = \{3, -1, 0\}$. What is the instantaneous rate of change of temperature measured by the particle when it is at the point $(0, 1, 2)$?