PHYS 301
HOMEWORK #12

Due : 22 April 2015

1. Bessel functions of the first kind, denoted as $J_n(x)$ are solutions to Bessel's differential equation, and appear in many contexts including electromagnetic theory and diffraction theory. The generating function for these functions is:

$$g(x, t) = e^{\frac{x}{2}\left(t - \frac{1}{t}\right)} = \sum_{-\infty}^{\infty} J_n(x)\, t^n$$

Use this generating function to establish these two recursion relations :

a) $J_{n-1}(x) + J_{n+1}(x) = \frac{2n}{x} J_n(x)$

b) $J_{n-1}(x) + J_{n+1}(x) = 2 J'_n(x)$

where $J'_n(x) = \frac{d(J_n(x))}{dx}$

c) Use the *Mathematica* BesselJ function to find expressions for for $J_{1/2}(x)$ and $J_{-1/2}(x)$. Then use the appropriate recursion relation to find an expression for $J_{3/2}(x)$ and compare your answer with the *Mathematica* expression.

(10 pts each for parts a and b; 5 pts for part c)

2. Find Legendre series for the following, writing out the first three non zero terms for each. You may leave your answers in terms of Legendre polynomials. (10 pts each part)

a) $f(x) = \begin{cases} x, & -1 < x < 0 \\ 0, & 0 < x < 1 \end{cases}$

b) $f(x) = 3x^2 + x - 1$

3. p. 626 no. 1