PHYS 301
HOMEWORK #7

Due : 20 March 2015

1. Verify the divergence theorem for the position vector $\mathbf{r}$:

$$\mathbf{r} = x\,\hat{\mathbf{x}} + y\,\hat{\mathbf{y}} + z\,\hat{\mathbf{z}}$$

over the sphere of radius R centered on the origin. (Hint : What is the unit normal to the surface of a sphere?)

2. Use the divergence theorem to evaluate

$$\int\int_S \left(x^2\,dy\,dz + y^2\,dx\,dz + z^2\,dx\,dy\right) \tag{1}$$

where S is the unit cube $0 \le x \le 1$, $0 \le y \le 1$, $0 \le z \le 1$

3. Verify Stokes' theorem for the vector :

$$\mathbf{v} = z\,\hat{\mathbf{x}} + x\,\hat{\mathbf{y}} \tag{2}$$

for the area defined by the unit square $0 \le x \le 1$, $0 \le y \le 1$.

4. Verify Stokes' theorem for the vector

$$\mathbf{v} = z^2\,\hat{\mathbf{x}} + x^2\,\hat{\mathbf{y}} + y^2\,\hat{\mathbf{z}} \tag{3}$$

over the same area in question 3.

5. Verify Stokes' theorem for the vector

$$\mathbf{v} = y\,\hat{\mathbf{x}} + x\,z^3\,\hat{\mathbf{y}} - z\,y^3\,\hat{\mathbf{z}} \tag{4}$$

over the circular disk $x^2 + y^2 \le 4$ in the plane z = -3.