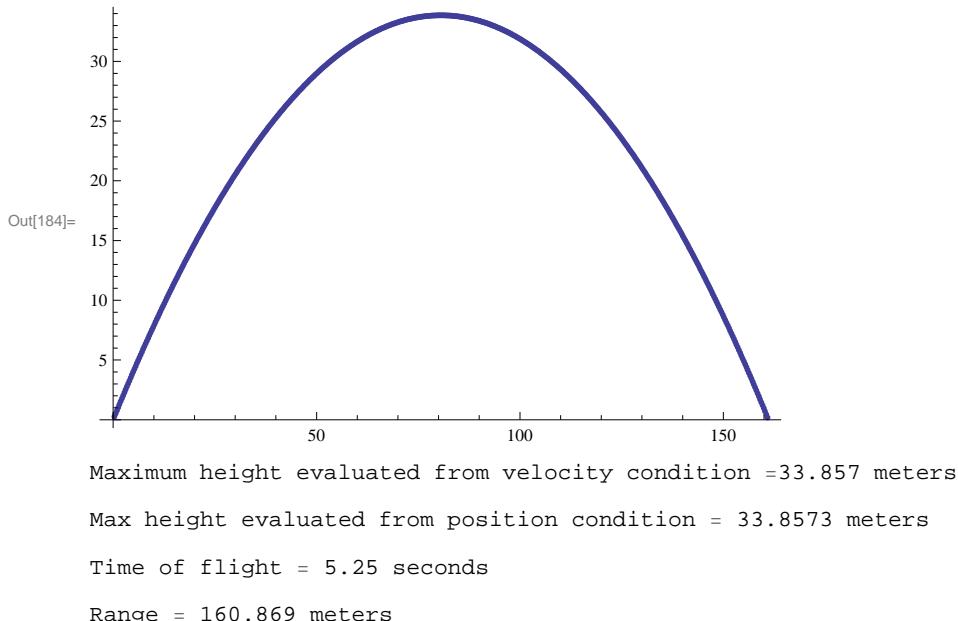


HOMEWORK #8-- Solutions

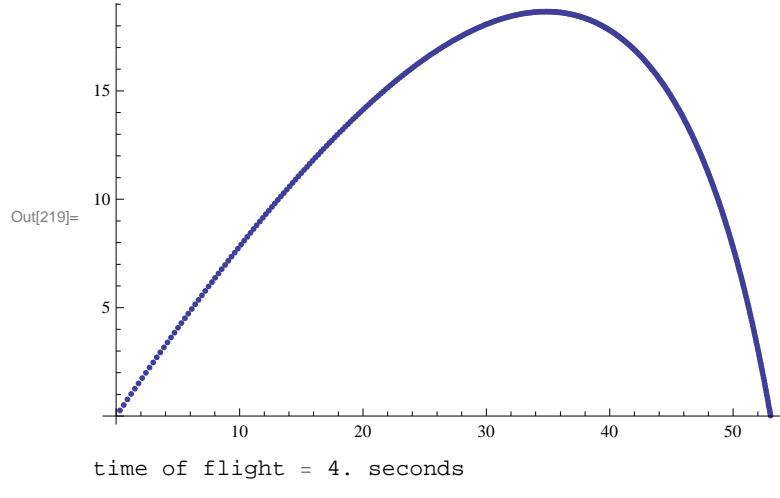
```
In[176]:= (* No friction case *)
Clear[x, y, vx, vy, h, g, nterms, nofrictioncase]
x[0] = 0; y[0] = 0; g = 9.8; h = 0.01; v0 = 40; θ = 2 π / 9;
vx[0] = v0 Cos[θ]; vy[0] = v0 Sin[θ];
vy[n_] := vy[n] = vy[n - 1] - g h
vx[n_] := vx[n] = vx[n - 1]
y[n_] := y[n] = y[n - 1] + vy[n - 1] h
x[n_] := x[n] = x[n - 1] + vx[n - 1] h

nterms = Catch[Do[If[y[n] < 0, Throw[n - 1]], {n, 1000}]];
nofrictioncase = ListPlot[Table[{x[n], y[n]}, {n, nterms}]]
maxht = Catch[Do[If[vy[n] < 0, Throw[y[n - 1]]], {n, nterms}]];
Print["Maximum height evaluated from velocity condition =", maxht, " meters"]
(* Maxht determines maximum height by determining where vy<0 *)
maxht2 = Catch[Do[If[y[n] < y[n - 1], Throw[y[n - 1]]], {n, nterms}]];
(* maxht2 determines max ht by finding where y[n] values begin to decrease *)
Print["Max height evaluated from position condition = ", maxht2, " meters"]
Print["Time of flight = ", nterms h, " seconds"]
Print["Range = ", x[nterms], " meters"]
Print[""]
```



```
In[210]:= (* With friction *)
Clear[axf, ayf, vx, vy, xf, yf, ntermsf, h, k, g, frictioncase, vxgraph, vygraph]
xf[0] = 0; yf[0] = 0; v0 = 40; θ = 2 π / 9; vx[0] = v0 Cos[θ];
vy[0] = v0 Sin[θ]; h = 0.01; k = 0.5; g = 9.8; m = 1;
axf[vx_] := axf[vx] = - (k / m) vx
ayf[vy_] := ayf[vy] = - (k / m) vy - g
vy[n_] := vy[n] = vy[n - 1] + ayf[vy[n - 1]] h
vx[n_] := vx[n] = vx[n - 1] + axf[vx[n - 1]] h
yf[n_] := yf[n] = yf[n - 1] + h vy[n - 1]
xf[n_] := xf[n] = xf[n - 1] + vx[n - 1] h
ntermsf = Catch[Do[If[yf[n] < 0, Throw[n - 1]], {n, 1000}]];
frictioncase = ListPlot[Table[{xf[n], yf[n]}, {n, ntermsf}]]
maxht = Catch[Do[If[vyf[n] < 0, Throw[yf[n - 1]]], {n, 1000}]];
Print["time of flight = ", ntermsf h, " seconds"]
Print["Max ht = ", maxht, " meters"]
Print["Range = ", xf[ntermsf], " meters"]

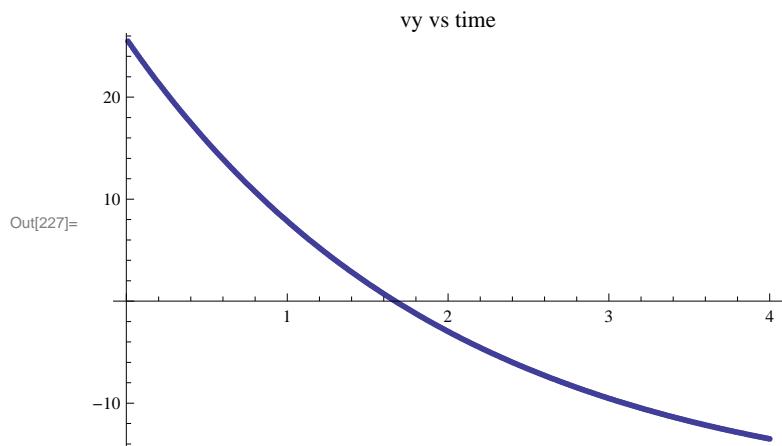
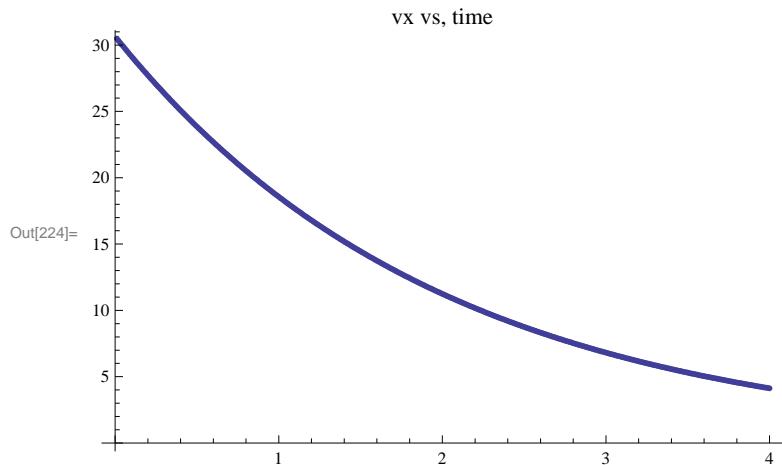
vxgraph = ListPlot[Table[{n h, vx[n]}, {n, ntermsf}], PlotLabel → "vx vs, time"]
(* vxgraph plots the velocity in the x direction as a function of time *)
Print["    "]
Print["    "]
vygraph = ListPlot[Table[{n h, vy[n]}, {n, ntermsf}], PlotLabel → "vy vs time"]
(* vygraph plots the velocity in the y direction as a function of time *)
```



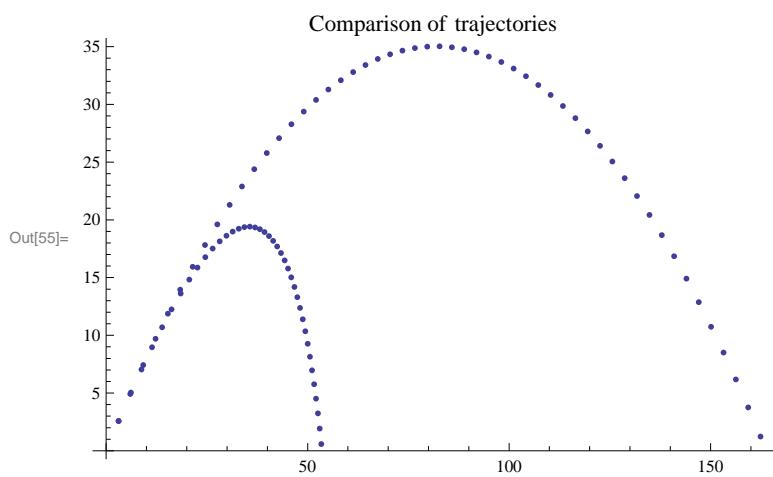
time of flight = 4. seconds

Max ht = 18.6543 meters

Range = 53.0312 meters



```
In[55]:= Show[nofrictioncase, frictioncase, PlotLabel -> "Comparison of trajectories"]
Print["  "]
Print["  "]
```



Now, let's look a bit more at the graphs for velocity as a function of time. For the case of linear friction that we have here, we can solve these equations (and the equations for $x(t)$ and $y(t)$) analytically. Let's start with the differential equation for velocity in the x direction. Using Newton's second law in the x direction, we have :

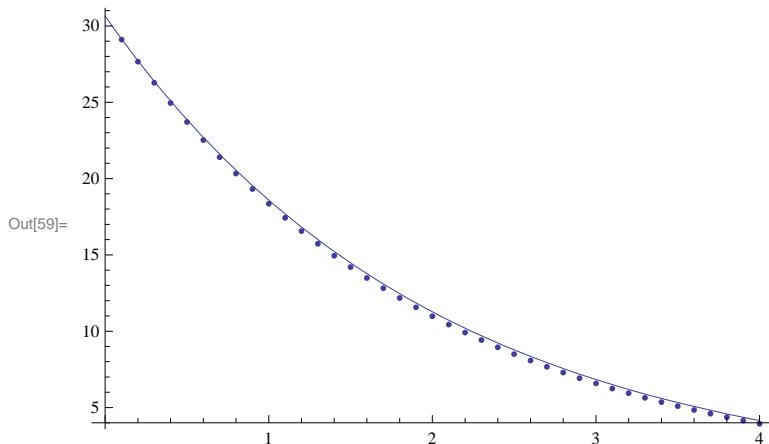
$$m \frac{dv_x}{dt} = -k v_x$$

Using elementary techniques of solving first order differential equations, you obtain :

$$v_x(t) = v_x(0) e^{-kt/m}$$

where $v_x(0)$ is the initial x velocity, which here is $40 \cos(40^\circ)$. Let's superimpose plots of $v_x(t)$ computed using this equation with the tabulated values of v_x determined from our Euler's method code:

```
In[58]:= vxtheoretical = Plot[v0 Cos[\theta] Exp[-k t / m], {t, 0, 4}];
Show[vxtheoretical, vxgraph]
```



And you can see excellent agreement. We do the same below and compare the tabulated vs. theoretical results for the y component of velocity :

```
In[67]:= vytheoretical = Plot[(m / k) (((k v0 + m g) / m) Exp[-k t / m] - g), {t, 0, 4}];
Show[vytheoretical, vygraph]
```

