# PHYS 301 <br> HOMEWORK \#1 

## Due : 22 Jan 2016

Homework is due in class on the due date noted. Review carefully the syllabus for proper format (complete, legible solutions; write on only one side of the paper, staple multiple sheets).

1. The transformation equations between the Cartesian and spherical polar coordinate systems are :

$$
\begin{aligned}
\mathrm{x} & =\mathrm{r} \sin \theta \cos \phi \\
\mathrm{y} & =\mathrm{r} \sin \theta \sin \phi \\
\mathrm{z} & =\mathrm{r} \cos \theta
\end{aligned}
$$

where r is the distance from the origin, $\theta$ is the polar angle and $\phi$ is the azimuthal angle. Use these equations to determine expressions for the total differentials dx , dy and dz in terms of $\mathrm{r}, \theta, \phi, \mathrm{dr}, \mathrm{d} \theta$ and $\mathrm{d} \phi$.
2. The Pythagorean theorem can be written in terms of the incremental distance between any two points in space as :

$$
\mathrm{ds}^{2}=\mathrm{dx}^{2}+\mathrm{dy}^{2}+\mathrm{dz}^{2}
$$

where ds is the differential distance between any two points. Use the expressions for $\mathrm{dx}, \mathrm{dy}$ and dz above to write $\mathrm{ds}^{2}$ in terms of spherical polar coordinates. What are the coefficients of the mixed terms ( $\mathrm{dr} \mathrm{d} \theta, \mathrm{dr} \mathrm{d} \phi$ and $\mathrm{d} \theta \mathrm{d} \phi$ )?
3. Consider the function

$$
\mathrm{f}=\mathrm{x}^{2}+\mathrm{y}^{2}+\mathrm{z}^{2}
$$

Find :
a) $\nabla \mathrm{f}(\operatorname{grad} \mathrm{f})$
b) $\nabla \cdot(\nabla \mathrm{f})(\operatorname{div} \operatorname{grad} \mathrm{f})$
c) $\nabla \times(\nabla \mathrm{f})(\operatorname{curl} \operatorname{grad} \mathrm{f})$
4. Compute the line integral for the function

$$
\mathbf{f}=-y \hat{\mathbf{x}}+x \hat{\mathbf{y}}
$$

along the circle of radius 3 centered on the origin. The symbols $\hat{\boldsymbol{x}}$ and $\hat{\boldsymbol{y}}$ indicate unit vectors in the x and y directions respectively.

