PHYS 301 HOMEWORK #1

Due : 22 Jan 2016

Homework is due in class on the due date noted. Review carefully the syllabus for proper format (complete, legible solutions; write on only one side of the paper, staple multiple sheets).

1. The transformation equations between the Cartesian and spherical polar coordinate systems are :

$$x = r \sin \theta \cos \phi$$

$$y = r \sin \theta \sin \phi$$

$$z = r \cos \theta$$

where r is the distance from the origin, θ is the polar angle and ϕ is the azimuthal angle. Use these equations to determine expressions for the total differentials dx, dy and dz in terms of r, θ , ϕ , dr, d θ and d ϕ .

2. The Pythagorean theorem can be written in terms of the incremental distance between any two points in space as :

$$ds^2 = dx^2 + dy^2 + dz^2$$

where ds is the differential distance between any two points. Use the expressions for dx, dy and dz above to write ds² in terms of spherical polar coordinates. What are the coefficients of the mixed terms (dr d θ , dr d ϕ and d θ d ϕ)?

3. Consider the function

$$f = x^2 + y^2 + z^2$$

Find :

a) ∇f (grad f)

b) $\nabla \cdot (\nabla f)$ (div grad f)

c) $\nabla \times (\nabla f)$ (curl grad f)

4. Compute the line integral for the function

$$\mathbf{f} = -\mathbf{y}\,\mathbf{\hat{x}} + \mathbf{x}\,\mathbf{\hat{y}}$$

along the circle of radius 3 centered on the origin. The symbols \hat{x} and \hat{y} indicate unit vectors in the x and y directions respectively.