PHYS 301
HOMEWORK #11

Due : 13 April 2016

1. Use series solution techniques to solve

$$\left(1+x^2\right) y'' - 2\, x\, y' + 2\, y = 0$$

Find the recursion relation, expressions for the coefficients, and write the solution as a power series.

2. Problem 12.56, page 672. Find the coefficients and write the series solution out to the x^5 term. (You will need to expand the $\sqrt{x+1}$ term as a Maclaurin series as shown in the book in section 12.4.

3. For this problem, refer to the discovery exercise in section 2.1 of the copy of Chapter 14 of the text that I emailed you earlier this week. Write a short Mathematica program that will compute the value of the integral :

$$\int_0^1 \left[\left(\frac{dy}{dx}\right)^2 + 10\, x\, y\right] dx$$

where y = x^n for $0 \le n \le 5$. Plot the results (the values of the integrals) on one set of axes to show the value of n that allows x^n to minimize this result. (ListLinePlot will show the data connected by lines, so you might find that format valuable.) Remember, a program is a coherent set of instructions that takes input and yields output, so there will be no credit for solutions in which you individually or manually compute these integrals. Use loop controls as needed to write as compact and elegant code as possible. Now that you are gaining some fluency in *Mathematica*, part of your grade will be based on the efficiency and elegance of your code. (20 pts for this question)

4. Consider an object of mass m dropped from rest from a height of 200 m above the surface of the Earth. In this problem, we will assume that there is air friction represented by f = - c v where c is a constnat and v is the velocity.

Adopt a coordinate system in which down is positive, so that your initial values are y(0)=0 and v(0)=0.

a) Write Newton' s second law for this object.

b) Write a short Mathematica program that will determine the height and velocity of this object during its flight. Use your program to determine the time of flight and velocity upon hitting the ground. For this program, assume first c = 0.2 and set the mass = 1 kg (why do you need to assign a mass in this problem?)

c) Compare your answers in b to the answers you would obtain in the case of no friction. Change the values (not the code) in your program to verify that your program yields the answers you obtain from elementary physics calculations.

d) What would be the time of flight and final velocity if the mass were 10 kg, 100 kg? Explain physically why increasing the mass causes these values to change in the direction they do.

e) Using your expression for Newton' s second law, determine the terminal velocity of a 2 kg mass. Use your program to determine from what height you would need to drop a 2 kg mass to reach its terminal velocity?