PHYS 301 HOMEWORK #3

Due : 8 Feb. 2016

On this homework assignment, you may use Mathematica to verify the results of integration, but you must do all integrals by hand and show all work for them (and for all other problems, of course).

1. Show that making the substitution u = 1/r transforms the differential equation :

$$\ddot{r} - \frac{h^2}{r^3} = \frac{-GM}{r^2}$$

to the differential equation :

$$\frac{\mathrm{d}^2\,\mathrm{u}}{\mathrm{d}\phi^2} + \mathrm{u} = \frac{\mathrm{G}\,\mathrm{M}}{\mathrm{h}^2}$$

2. In our study of Fourier series, we will be very interested in evaluating integrals of the form :

 $\int_{-\pi}^{\pi} \cos(mx) \cos(nx) dx \qquad \int_{-\pi}^{\pi} \cos(mx) \sin(nx) dx \qquad \int_{-\pi}^{\pi} \sin(mx) \sin(nx) dx$ where m and n are integers.

Start with the trigonometric addition and subtraction identities $(\sin (m \pm n) x \text{ and } \cos (m \pm n) x)$ and find the values of each integral above. Make sure you consider separately the cases where m = nand m != n.

3. In our study of Fourier series, we will also learn to write trig functions in terms of exponentials by using Euler's formula :

$$e^{\pm ix} = \cos x \pm i \sin x$$

a) Use Euler's formula to express cos x and sin x in terms of exponentials.

b) Remembering the symmetry of trig functions (cos is even, sin is odd), show that :

 $\int_{-\pi}^{\pi} e^{i k x} dx = 0 \text{ for all integer values of } k$

4. Use your results from above to evaluate the integrals

 $\int_{-\pi}^{\pi} \sin(mx) \sin(nx) dx \text{ and } \int_{-\pi}^{\pi} \cos(mx) \cos(nx) dx$

where m and n are integers Again, be sure to consider separately cases where m and n are equal and unequal.

5. Problem 9.34 from text; all parts. Use Mathematica to produce your plots and submit your plots

with your homework assignment.