PHYS 301 HOMEWORK #5

Due : Friday 19 Feb. 2016

You may use *Mathematica* to compute Fourier coefficients; if you do, submit your *Mathematica* output with your homework. (But you may not compute Fourier series by using the Fourier series function in *Mathematica*)

1. In class we computed the Fourier series for a guitar string plucked in the middle. Consider a guitar string of length L plucked at L/4 as shown in the diagram below :



Note the scale of the axes; I am using the parameters from the book (L = 65 cm, h = 3 cm). Make the odd extension to this function and compute the Fourier series. Write out the first five non zero terms of the Fourier expansion. (Do your integrals and report your answer in general terms; in other words, solve your integral using L and h (don't use numerical values in your integrals, although you will need to use numerical values for L and h if you wish to plot them.)

2. The function below responds a sound wave propagating through the atmosphere. At a given location, the pressure varies with time according to :

$$p(t) = \begin{cases} 7/8, & -1/524 < t < -1/1048 \\ -1, & -1/1048 < t < 0 \\ 1, & 0 < t < 1/1048 \\ -7/8, & 1/1048 < t < 1/524 \\ 7/8, & 1/524 < t < 3/1048 \\ -1, & 3/1048 < t < 1/262 \\ 1, & 1/262 < t < 5/1048 \end{cases}$$

the pressure is measured in units of 10^{-6} atm, time is measured in seconds. Draw a graph of this function and determine if it is even or odd (it is one of those choices). Explain why you do not have to make any extension (even or odd) to this function to compute its Fourier series. Compute its Fourier series and write out the first five non zero terms of the expansion. Which frequency will be loudest? What is the ratio of the amplitude of the loudest frequency to the next loudest frequency?



Current is plotted on the vertical axis, time on the horizontal. The discontinuity occurs at t = 1/120s, and this graph shows the value of current for the interval [0,1/60]s. Assume that this part of the graph is repeated 60 times per second. (Think carefully about your chosen value of L). Find the Fourier series representing this function and write out explicitly the first five non zero terms of the expansion. (This is called a rectified sawtooth wave.)

4. Find the Fourier series for the function :

f (x) = x²,
$$\frac{-1}{2} < x < \frac{1}{2}$$

Write out the first five non zero term sof the expansion.