PHYS 301 HOMEWORK #9

Due : 1 April 2016

1. Consider a scalar function ϕ (x, y, z).

a) What is the gradient of this function? (5)

b) Find the numerical value of the curl of the gradient by computing the relevant determinant. (5)

c) What property of partial derivatives allows you to compute this numerical value? (5)

d) Use Einstein summation notation to prove that this result $(\nabla \times (\nabla \phi))$ is general for all scalar function. (10).

2. Use Einstein summation notation to show that :

$$(\mathbf{A} \times \mathbf{B}) \cdot (\mathbf{C} \times \mathbf{D}) = (\mathbf{A} \cdot \mathbf{C}) (\mathbf{B} \cdot \mathbf{D}) - (\mathbf{B} \cdot \mathbf{C}) (\mathbf{A} \cdot \mathbf{D})$$

3. Verify the divergence theorem for

$$\mathbf{v} = (\mathbf{x} \mathbf{y}) \,\mathbf{\hat{x}} + (2 \mathbf{y} \mathbf{z}) \,\mathbf{\hat{y}} + (3 \mathbf{z} \mathbf{x}) \,\mathbf{\hat{z}}$$

in the volume defined by the cube with sides of length two with vertices at (0, 0, 0) and (2, 2, 2).

4. 8.162 page 436

6. a) Use discretization and recursion techniques to solve numerically the differential equation :

$$\frac{\mathrm{d}y}{\mathrm{d}x} = x\cos^2 y$$

with initial conditions x[0] = 0 and $y[0] = \pi/4$. Use the ListPlot command to plot the solution between x = 0 and x = 10.

b) Use separation of variables to solve this equation by hand (do not use any Mathematica library routines like DSolve) with these initial conditions. Plot the solution on the same set of axes as the numerical solution and show the two methods yield the same solution.

7. Refer to the Wikipedia webpage for Computus. This should take you to a page describing various calendar calculations. Scroll down to the section "Anonymous Gregorian algorithm." The table provides a step by step algorithm to compute the date of Easter as celebrated in the Catholic Church and other Western Christian religions. (Greek and Russian Orthodox among others use a different algorithm and often celebrate Easter on a different day).

Use the *Mathematica* functions Mod, Floor and RandomInteger to choose a random date between 1800 and 2100 and compute the date of Easter for that year. Your program should output the year chosen, and the computed month and date of Easter. (To check your program, make sure it gives the correct answer for this year).