

PHYS 301

HOMework #3

Due : 6 Feb. 2017

1. Using results from the last homework assignment, find expressions for velocity and acceleration in the spherical polar coordinate system.
2. Consider the function expressed in spherical polar coordinates :

$$\mathbf{f} = \frac{1}{r} \hat{\mathbf{r}}$$

Write this function completely in terms of Cartesian coordinates and Cartesian unit vectors.

3. Consider the function expressed in Cartesian coordinates :

$$\mathbf{f} = 2y \hat{\mathbf{x}} - x \hat{\mathbf{y}}$$

Write this function completely in plane polar coordinates, and then use plane polar coordinates to compute the work done in moving this force along the circle $\rho = 2$ from $\phi = 0$ to $\phi = \pi$. (Make sure you convert both the function and $d\mathbf{l}$ into plane polar coordinates).

4. The transformation equation for elliptic cylindrical coordinates (u, v, z) are :

$$x = a \cosh u \cos v$$

$$y = a \sinh u \sin v$$

$$z = z$$

where a is a constant and \cosh and \sinh are the hyperbolic cos and sin functions. If you are not familiar with them, read up on them making sure you know their definitions, derivatives, and properties.

Determine whether this coordinate system is orthogonal; find the scale factors h_u , h_v , and h_z .

5. In class we stated that setting $u = 1/r$ transforms the differential equation :

$$\ddot{\mathbf{r}} - \frac{h^2}{r^3} = \frac{-GM}{r^2} \quad (1)$$

into the almost trivial:

$$\frac{d^2 u}{d\phi^2} + u = \frac{GM}{h^2} \quad (2)$$

make the required substitution into equation (1) and show that you obtain eq. (2). You will need to make successive use of the chain rule to transform the second derivative term.