PHYS 301
HOMEWORK #6

Due : 20 March 2017

1. Show that (or justify that) :

$$\delta_{ij}\,\delta_{jk} = \delta_{ik}$$

Where the δ are Kronecker Deltas.

2. The Levi - Civita permutation tensor can be defined in any number of dimensions, and has the familiar properties that :

$$\epsilon_{ijk\ldots rst} = \begin{cases} 0, & \text{if any two indices are the same} \\ 1, & \text{if all indices are different and are an even permutation} \\ -1, & \text{if all indices are different and are an odd permuation} \end{cases}$$

Consider then the product of these five dimensional permutation tensors:

$$\epsilon_{ijklm}\,\epsilon_{ijklm}$$

What is the value of this product? You should obtain a numerical result; show your work and/or explain your reasoning.

For problems 3 - 5, use Einstein summation notation. No credit will be given for using term - by - term component expansions.

3. Prove that $\nabla \times (\mathbf{A} \times \mathbf{B}) = \mathbf{A}\,(\nabla \cdot \mathbf{B}) - \mathbf{B}\,(\nabla \cdot \mathbf{A}) + (\mathbf{B} \cdot \nabla)\,\mathbf{A} - (\mathbf{A} \cdot \nabla)\,\mathbf{B}$

4. Prove that $\nabla \cdot (\mathbf{A} \times \mathbf{B}) = \mathbf{B} \cdot (\nabla \times \mathbf{A}) - \mathbf{A} \cdot (\nabla \times \mathbf{B})$

5. If f and g are both differentiable functions of {x, y, z}, use summation notation to derive the identity for $\nabla\left(\frac{f}{g}\right)$

6. For this problem, you will use both term by term component expansions and summation notation to determine the value of $\nabla\times(\nabla\phi)$ where ϕ is a scalar function.

a) Write the gradient of ϕ explicitly in Cartesian coordinates and compute its curl. What property of partial differentiation did you use to determine a numerical value for this expression?

b) Now, use summation notation to evaluate $\nabla\times(\nabla\phi)$.