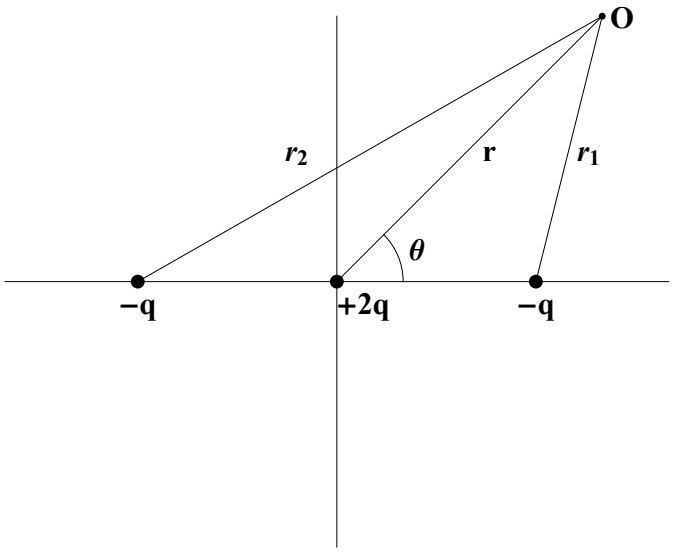
PHYS 301
HOMEWORK #9

Due : 12 April 2017

1. Consider the electric quadrupole in the diagram below :



The -q charges are located at (a,0) and (-a,0) along the x-axis. Express the potential at O due to this arrangement of charges in terms of Legendre polynomials. Assuming a<<r, what is the leading term of the expansion?

2. Determine the Legendre coefficients out to c_5 for the function:

$$f(x) = \begin{cases} -1, & -1 < x < 0 \\ 1, & 0 < x < 1 \end{cases}$$

and write out the Legendre series for this function.

3. Expand the following as Legendre series; you may use Mathematica to verify your results, but you must show all integration by hand. (10 pts for each series.

a) $x^2 - x$

b) $7x^4 - 3x + 1$

4. The generating function for Bessel' s functions of the first kind is :

$$g(x, t) = e^{(x/2)(t-1/t)} = \sum_{n=-\infty}^{\infty} J_n(x)\, t^n$$

where J_n is the nth order Bessel function. Use the generating function to show that:

a) $J_{n-1}(x) + J_{n+1}(x) = \frac{2n}{x} J_n(x)$

b) $J_{n-1}(x) + J_{n+1}(x) = 2 \frac{dJ_n(x)}{dx}$

10 pts for each part.