PHYS 314
FINAL EXAM QUESTIONS

Below are candidate questions for the final exam. The final exam will consist of a subset of these questions. To save typing below, I might refer you to equations in the book or refer to previous homework assignments. If the particular question appears on the final, I will write the question out completely so that you won't have to guess what it refers to.

You may work in groups and consult any other sources you wish, however, I will not provide guidance or assistance on any of these questions. You may not bring any notes to the final exam, but I will provide any and all equations you need to solve these.

1. A and B are invertible matrices. Use matrix operations to show that $(AB)^{-1} = B$^$-1\, A^{-1}$

2. A and B are square matrices of the same dimension. The trace of a matrix is the sum of its diagonal elements. Use summation notation to show that Trace (AB) = Trace (BA)

3. A uniform rod of weight W is supported by two vertical props at each end. At t = 0 one of these supports is kicked out. Find the force on the other support immediately thereafter.

4. An inclined plane makes an angle α with the horizontal. A projectile is fired from the bottom of the plane with speed v_o in a direction making an angle β with the horzon ($\beta > \alpha$). Neglect friction in this problem.

a) Prove the range up the incline can be expressed as

$$R = \frac{2\, v_o^2 \sin(\beta - \alpha) \cos\beta}{g \cos^2 \alpha}$$

b) Prove the maximum range up the incline is

$$R_{max} = \frac{v_o^2}{g\,(1 + \sin\alpha)}$$

5. A projectile is launched on a level surface with inital velocity v_o at an angle α to the horizontal. Assuming there is a frictional force proportional to $\beta\, \mathbf{v}$, find expressions for the time to maximum height, and the maximum height.

6. Refer to problem 3 of HW 1. Assume the mass has an initial speed of v_o at the top of the sphere. Find the angle at which the particle leaves sphere assuming that $v_o \leq \sqrt{gR}$.

7. A weight W is suspended from three equal strings of length l which are attached to the three vertices of a horizontal equilateral triangle of side s. Find the tension in the strings.

8. A particle of mass 5 grams moves along the x axis under the influence of two forces; a) a force of attraction to the origin which in dynes is numerically equal to 40 times the instantaneous distance from O, and b) a damping force proportional to the instantaneous speed such that when the speed is 10 cm/s the damping force is 200 dynes.

Assuming the particle starts from rest a distance of 20 cm from the origin,

a) set up the differential equation and conditions describing the motion,

b) the position of the particle at any time

c) its amplitude and period

d) the logarithmic decrement

9. A cylinder with its axis vertical floats in a liquid of density ρ. It is pushed down slightly and released. Find the period of oscillation if the cylinder has cross sectional area A and mass m. (Hint : the restoring force is buoyancy).

10. Find the force of attraction of a think uniform rod of length a and mass M on a mass m which lies outside the rod but on the same line as the rod and distance b from an end.

11. A hemisphere of mass M and radius a has a mass m located at its center. Find the force of attraction if the hemisphere is i) a thin shell, ii) solid,

12. Solve HW1 problem 3 using Lagrangian mechanics.

13. Solve HW1 problem 5 using Lagrangian mechanics.

14. Consider a mass m on a spring of spring constant k and natural length L. The mass can swing from side to side (as well as stretch or compress the spring). Choose a set of generalized coordinates and write the Lagrangian, find the equations of motion and the equilibria (configurations where time derivatives are zero).

15. A particle of mass m moves under the influence of gravity on the inner surface of the paraboloid of revoutons $x^2 + y^2 = a\,z$

where a is a constant. Use Lagrangian mechanics to find the equations of motion.

16. Refer to HW9 problem 2. Show that the substitution u = cos (θ/2) results in the differential equation :

$$\ddot{u} + \frac{g}{4\,a} u = 0$$

Find the solution to this equation, and find the period of oscillation of the particle.

17. Two equal masses m, connected by a massless string, hang over two pulley of negligible size and mass. The left mass moves in a vertical line, but the right one is free to swing back and forth in

the plane of the masses and pulleys. Find the equations of motion for r and θ. (I will provide a diagram in class).