PHYS 380
WEEK OF SEPT 28

Let' s try some more elaborate substitutions to probe more deeply into the nature of ODEs and in particular, Bessel' s ODE.

1. Consider the ODE :

$$x^2\, y'' + 2\, x\, y' + \left[x^2 - n\,(n+1)\right] y = 0$$

a) Make the substitution :

$$v = x^s\, y$$

and substitute back into the equation. Get the correct expressions for dy/dx and for the second derivative, to convert this to an ODE in v. Part of the problem is to figure out the correct value of s.

b) Multiply through as needed to ensure that the leading term has the coefficient of x^2. Then choose the correct value of s so that the first derivative has a coefficient of x.

c) Identify the ensuing equation and write down the solution.

2. Consider the ODE :

$$x^2\, y'' + \left[x^2 - n\,(n+1)\right] y = 0$$

Follow the procedure of question 1 and find the solution to this ODE (do not use the method of Frobenius; make the substitutions until you get the ODE into a form you recognize).

3. Now the piece de resistance :

$$y'' + \frac{1-2\,a}{x}\, y' + \left[\left(b\, c\, x^{c-1}\right)^2 + \frac{a^2 - p^2\, c^2}{x^2}\right] y = 0$$

many ODEs are special cases of this, where a, b, c and p are constants. First make the substitution :

$$y\,(x) = x^a\, u\,(x)$$

to transform this into a second order ODE in u (x). Then, set

$$z = b\, x^c$$

to make this an ODE for u (z). Simplify as much as possible, and when this looks like something you recognize, write down the solution.(Review eqs. 16.1 and 16.2 on p. 593 of Boas).

These are good problems to work on in groups. Let' s see some group solutions next Thursday.