LOYOLA UNIVERSITY CHICAGO

Department of Mathematics and Statistics
Lake shore campus | 1032 W. Sheridan Road | Chicago, Illinois 60660
Phone 773.508.3558 | Fax 773.508.2123 | www.math.luc.edu

Math 118 - Fall 2024 - Common Final Exam, version A

Print name:____

Section number:__________ **Instructor's name:**______________________________

Directions:

- This exam has 14 questions. Please check that your exam is complete, but otherwise keep this page closed until the start of the exam is called.
- Fill in your name, and your instructor's name.
- It will be graded out of 100 points.
- Show your work. Answers (even correct ones) without the corresponding work will receive no credit.
- A formula sheet has been provided with this exam. You may not refer to any other notes during the exam.
- You may use a calculator which does not allow internet access. The use of any notes or electronic devices other than a calculator is prohibited.
- **Unless otherwise stated, round any constants to two decimal places if necessary.**

Good luck!

Question:	1	2	3	4	5	6	7	8
Points:	9	5	10	11	8	11	6	8
Score:								

Question:	9	10	11	12	13	14		Total
Points:	6	5	6	5	5	5		100
Score:								

1. (9 points) The revenue of Madeline's tutoring business is $10,000 in the year 2024. Recall that a linear function has a general form of $P = mt + b$ and an exponential function has a general form of $P = a \cdot b^t$.

   (a) If the revenue of Madeline's business is decreasing by $300 per year, find a formula for the function $R(t)$, the revenue t years after 2024.

   (b) If Madeline's revenue is instead increasing by 12% per year, find a formula for the function $R(t)$, the revenue t years after 2024.

   (c) Using your formula from part b, find the year that Madeline's revenue will hit $30,000$. Round to the nearest whole number.

2. (5 points) Find the half-life of a radioactive substance that decays at a continuous rate of 7% per hour.

3. (10 points) Determine if the following functions are linear, exponential, or neither. Then, find an equation for each function or explain why it cannot be done.

(a)

x	-1	0	1	2	3
$f(x)$	-7	-5	-3	-1	1

(b)

x	0	1	2	3	4
$g(x)$	3	6	12	24	48

4. (11 points) Elliot opens a bank account with an initial deposit of $\$20,000$. It earns interest at a nominal rate of 3% per year.

(a) Find the balance of their account after 5 years if interest is compounded as follows.

i. Annually (once per year).

ii. Quarterly (4 times per year).

iii. Continuously.

(b) Of the three accounts listed in part (a) of this question, which has the highest effective rate? That is, which account increases the most over the course of one year?

5. (8 points) Consider the exponential function $Q = 182(0.97)^t$.

(a) Determine if this function displays exponential growth or decay.
Circle one: **exponential growth** or **exponential decay**. Explain your answer in a sentence.

(b) Give the initial value for this function.

(c) Give the growth rate for this function. Write your answer as a percentage.

(d) Give the domain for the given function.

(e) Give the range for the given function.

(f) Write the given function in the form $Q = ae^{kt}$.

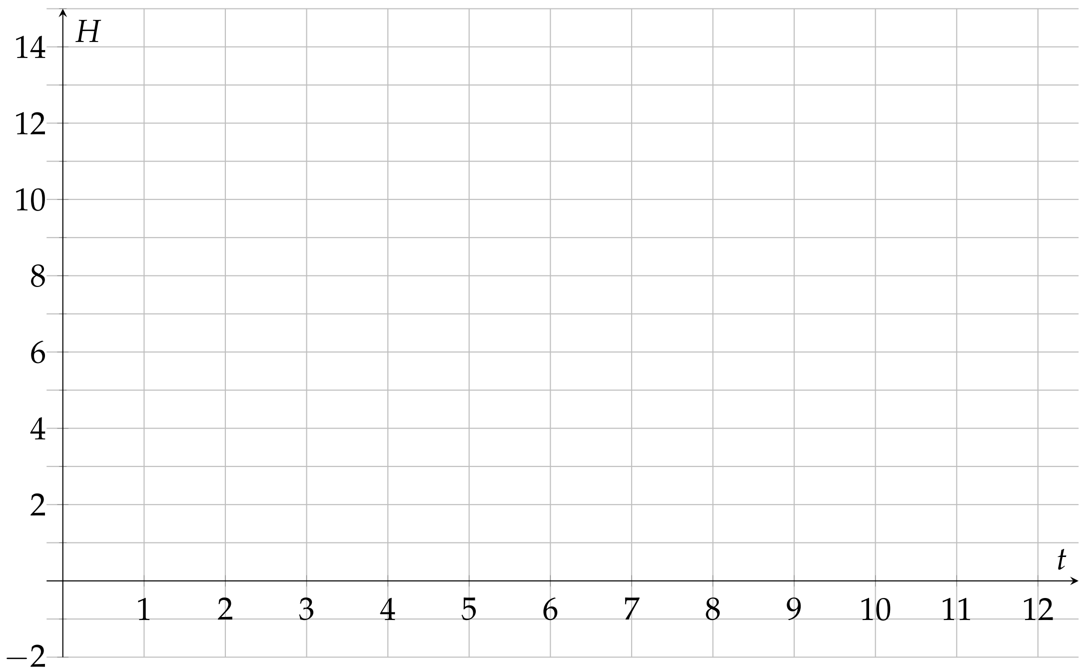
6. (11 points) The height of a spring oscillates sinusoidally between 3 feet at $t = 0$ seconds and 11 feet at $t = 5$ seconds. Consider the function, $H = f(t)$, which gives the height, H, as a function of time, t, in seconds.

(a) Find the amplitude, period, and midline of the function $H = f(t)$.

- The amplitude is ________
- The period is ________
- The midline is ________

(b) Find a formula for the population, $H = f(t)$.

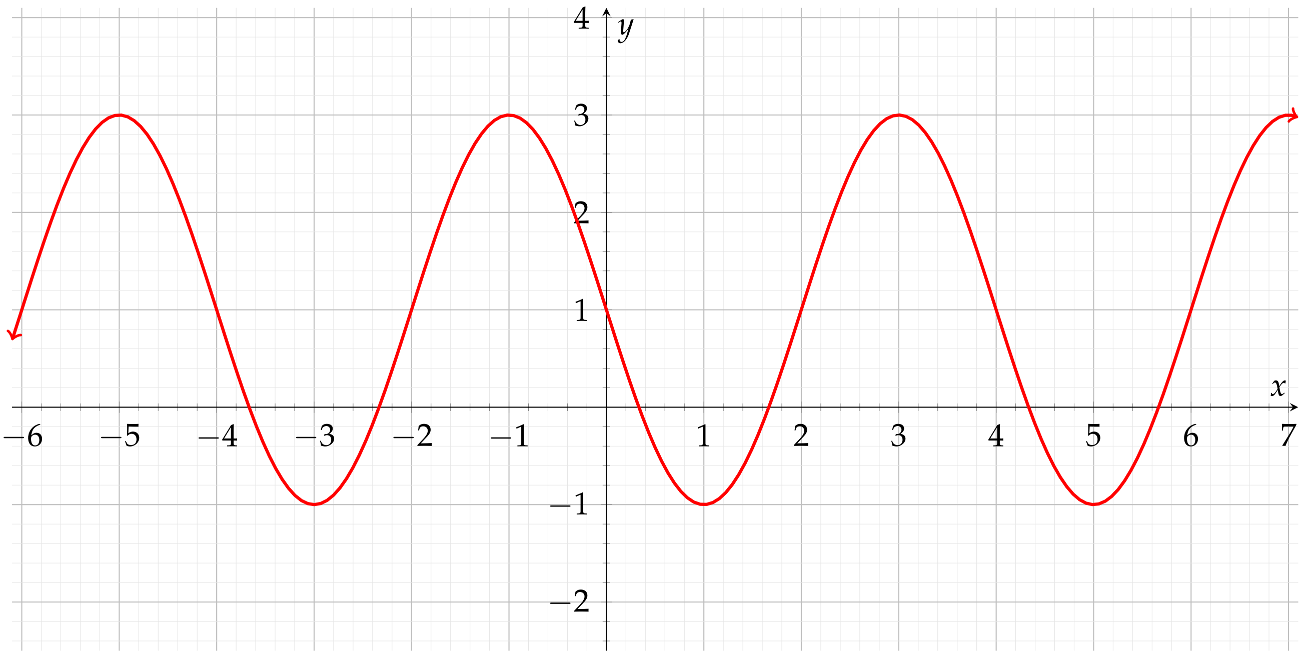
(c) Graph H as a function of t over the whole interval shown below.



(d) Write an equation whose solutions give all times that the height of the spring is 6 feet.

(e) Find a solution to the equation you found in part (d), giving your answer in terms of an inverse trig function and also evaluate with correct units.

7. (6 points) Find a formula of the trigonometric function shown in the graph below.



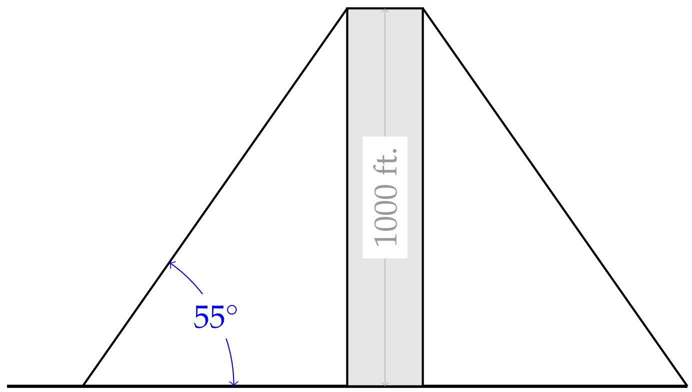
8. (8 points) For an angle α where $\pi < \alpha < \frac{3\pi}{2}$ such that $\sin(\alpha) = -\frac{3}{4}$, find the given quantities without finding α. Give an exact answer for each part.

(a) $\cos(\alpha)$

(b) $\tan(\alpha)$

(c) $\cos(\alpha - \frac{\pi}{4})$

9. (6 points) The top of a 1000 foot tower is to be anchored by cables that make an angle of 55° with the ground. A diagram of the situation is below.



(a) How long must the cables be?

(b) How far from the base of the tower should the anchors be placed?

10. (5 points) Let $f(x) = 2x - 5$, $g(x) = -4x + 3$ and $h(x) = \cos(x)$. Find the following, and simplify your answers completely:

(a) $f(g(2))$

(b) $h(g(f(x)))$

11. (6 points) Let $P = f(t) = 700(1.06)^t$ be the population of a town. Let t be measured in years since 2024.

   (a) Evaluate $f(8)$. Round to the nearest whole number. Describe in words what this quantity represents. Write your answer in a complete sentence with units.

   (b) Find a formula for $f^{-1}(P)$ in terms of P. Give an exact answer.

   (c) Evaluate $f^{-1}(1000)$. Round to the nearest whole number

   (d) Describe in words what the quantity you found in part (c) represents. Write your answer in a complete sentence with units.

12. (5 points) Decompose the function

$$f(x) = \frac{4}{\ln(3x+6)}$$

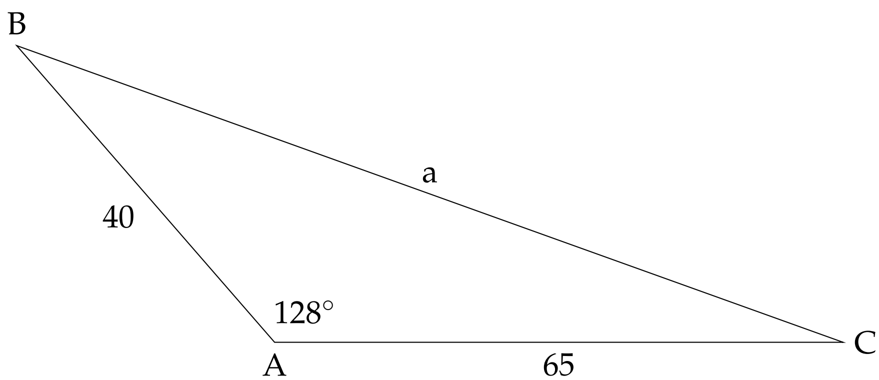
into a composition of two new functions u and v, where v is the inside function. That is $f(x) = u(v(x))$, so that $u(x) \neq x$ and $v(x) \neq x$.

13. (5 points) Perform the following conversions.

(a) Convert the Cartesian coordinates $(x, y) = (5, 5)$ to polar coordinates. Give an exact answer.

(b) Convert the polar coordinates $(r, \theta) = (7, \frac{2\pi}{3})$ to Cartesian coordinates. Give an exact answer.

14. (5 points) Find the length of the missing side, a, in the diagram below.



Exponential and Logarithm Formulas
Linear Function: $Q(t) = mt + b$
Exponential Function: $Q(t) = a \cdot b^t$
Continuous Exponential Function: $Q(t) = a \cdot e^{kt}$
Simple Interest: $B = P(1+r)^t$
Compound Interest: $B = P\left(1 + \frac{r}{n}\right)^{nt}$

Trigonometry Formulas
1 radian $= \frac{180}{\pi}$ degrees and 1 degree $= \frac{\pi}{180}$ radians

$$\sin(\theta) = \frac{opp}{hyp} = \frac{y}{r} \quad \cos(\theta) = \frac{adj}{hyp} = \frac{x}{r} \quad \tan(\theta) = \frac{opp}{adj} = \frac{y}{x} = \frac{\sin(\theta)}{\cos(\theta)}$$

$$\csc(\theta) = \frac{1}{\sin(\theta)} = \frac{r}{y} \quad \sec(\theta) = \frac{1}{\cos(\theta)} = \frac{r}{x} \quad \cot(\theta) = \frac{1}{\tan(\theta)} = \frac{x}{y} = \frac{\cos(\theta)}{\sin(\theta)}$$

Pythagorean Identities: $\sin^2(\theta) + \cos^2(\theta) = 1 \quad \tan^2(\theta) + 1 = \sec^2(\theta) \quad 1 + \cot^2(\theta) = \csc^2(\theta)$

Sum and Difference Formulas:
$\sin(A+B) = \sin(A)\cos(B) + \cos(A)\sin(B)$
$\sin(A-B) = \sin(A)\cos(B) - \cos(A)\sin(B)$
$\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B)$
$\cos(A-B) = \cos(A)\cos(B) + \sin(A)\sin(B)$

Even-Odd Identities: $\sin(-x) = -\sin(x)$ and $\cos(-x) = \cos(x)$ and $\tan(-x) = -\tan(x)$

Other identities: $\sin(\theta) = \sin(\pi - \theta)$, $\cos(\theta) = -\cos(\pi - \theta)$ and $\tan(\theta) = -\tan(\pi - \theta)$

General form for sine and cosine: $f(t) = A\sin(Bt) + k$ and $f(t) = A\cos(Bt) + k$

General form with horizontal shift: $f(t) = A\sin(B(t-h)) + k$ and $f(t) = A\cos(B(t-h)) + k)$

Period for sine and cosine: $P = \frac{2\pi}{|B|}$ or $PB = 2\pi$. Amplitude $= |A| = \frac{max - min}{2}$. Midline: $y = k$, where $k = \frac{max + min}{2}$

Law of Sines: $\frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c}$

Law of Cosines: $c^2 = a^2 + b^2 - 2ab\cos(C)$

Arc Length: $s = r\theta$

Inverse Trig Functions
$\theta = \cos^{-1}(y)$ provided that $y = \cos(\theta)$ and $0 \le \theta \le \pi$
$\theta = \sin^{-1}(y)$ provided that $y = \sin(\theta)$ and $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$
$\theta = \tan^{-1}(y)$ provided that $y = \tan(\theta)$ and $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$
Polar coordinates conversions
$r^2 = x^2 + y^2$, $\tan(\theta) = \frac{y}{x}$, $x = r\cos(\theta)$, $y = r\sin(\theta)$

The Unit Circle