



Department of Mathematics and Statistics

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Math 118 - Spring 2021 - Common Final Exam, version B

Directions:

- This exam has 16 questions. Please check that your exam is complete.
- You have two hours to complete this exam. It will be graded out of 116 points.
- You must be in the final exam zoom room for your own Math 118 class (communicated to you by your instructor) for proctoring. Exams from students not in the zoom proctor room will not be graded.
- You must remain visible in your camera's view while working on the exam.
- Please keep your microphone muted at all times. If you have any clarifying questions about the exam, or run into any technical trouble, please communicate with your instructor using Zoom chat.
- Show your work. Answers (even correct ones) without the corresponding work will receive no credit.
- This is an open-note exam, and you may use a calculator. However, you may not use any resources other than your own notes. For example, you may not look any websites, textbooks, or communicate with anyone other than your instructor while taking the exam.
- An exact answer means something like $\ln(2)$ rather than 0.6931 or π rather than 3.1415. Make sure to give exact answers where they are asked for.
- After you have finished your exam, convert it to a single pdf file and upload it to the test site in Sakai, or follow your instructor's directions for uploading.

Good luck!

Exam continues...

1. Consider the function $f(t) = \frac{50}{10 \cdot 2^{-t}}$.

(a) (4 points) Write the given function in the form $f(t) = ab^t$.

(b) (4 points) Give the initial value, growth factor, and growth rate for f .

2. A bank account is opened, it gains interest at a nominal annual rate of 5%, compounded monthly (that is 12 times per year). It currently has a balance of \$6000.

(a) (4 points) What will the balance be after 20 years? Give your answer as an exponential expression, and also give a value rounded to the nearest cent.

(b) (4 points) What is the effective rate (or APY) for this account? Give your answer as a percentage rounded to two decimal places, like $n.nn\%$ Explain in practical terms the meaning of the effective rate.

3. (6 points) Find the average rate of change of the function $f(x) = 6b^x$ over the interval from $x = 0$ to $x = 3$.

4. A radioactive isotope has a half-life of 23 days. A researcher puts a 90 g sample of the isotope in storage.

(a) (4 points) Write the expression for $Q(t)$, the quantity of the isotope remaining after t days.

(b) (4 points) Compute how long it will take until the sample is reduced to 10 g of the isotope. Give your answer rounded to one decimal place with correct units.

Exam continues...

5. Consider the exponential function $Q = -3.5(0.359)^t$.

(a) (2 points) Write the given function in the form $Q = ae^{kt}$.

(b) (3 points) Give the initial value, growth factor, and growth rate for the function.

(c) (2 points) Give the percentage rate of change and continuous rate of change for the function.

(d) (3 points) Determine if this function displays exponential growth or decay. Also, find the end behavior of the graph.

6. (6 points) Solve for x , giving an exact answer.

$$\ln\left(\frac{3}{e^{-x} + 2}\right) = 1$$

7. Determine which function dominates. *Justify.*

(a) (2 points) $f(x) = 10 \cdot x^2$ or $g(x) = 3 \cdot \log(4x)$.

(b) (2 points) $f(x) = 12,000 \cdot x^{120}$ or $g(x) = 0.2 \cdot e^{-0.2x}$.

(c) (2 points) $f(x) = 0.999 \cdot x$ or $g(x) = (0.999)^x$.

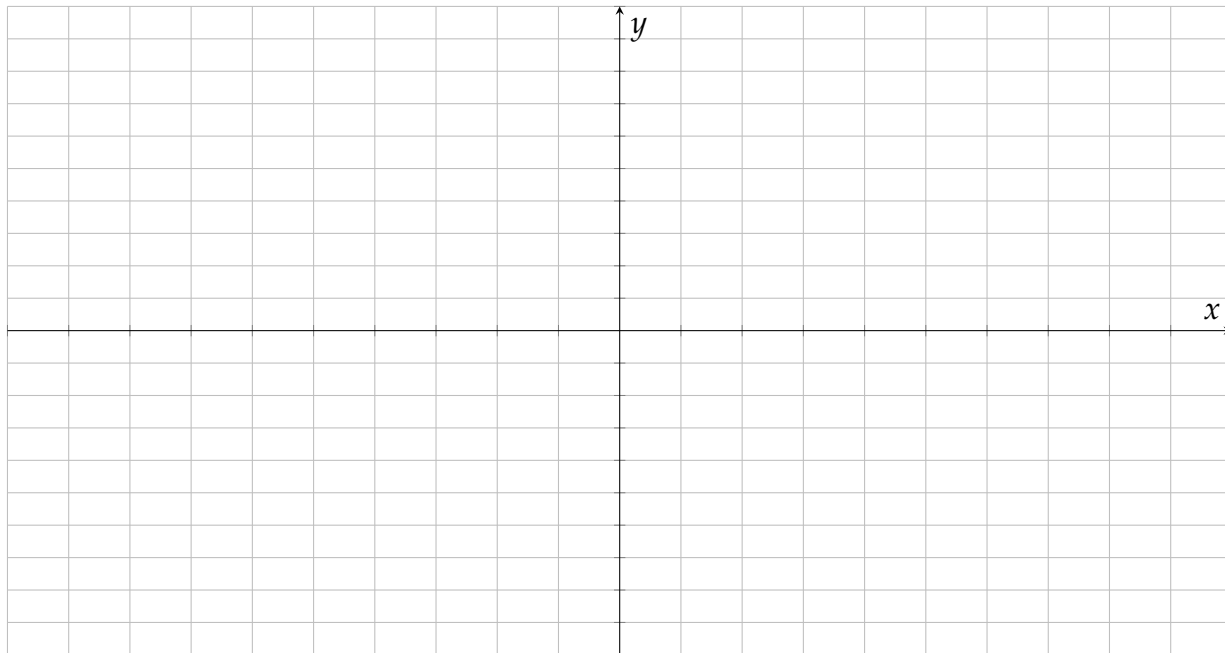
(d) (2 points) $f(x) = x^5$ or $g(x) = 5^{x-1}$.

Exam continues...

8. (8 points) Describe in a sentence the transformations of the graph of $f(x) = \cos(x)$ to arrive at the graph of

$$g(x) = 4 \cos \left(\pi \left(x + \frac{1}{4} \right) \right) - 3.$$

Graph $g(x)$ and label 4 points with (x, y) coordinates.



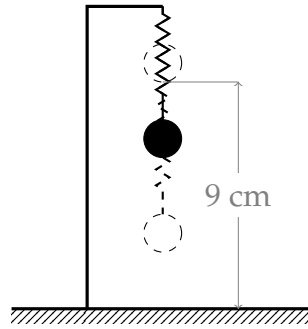
9. Identify the following:

(a) (4 points) The domain and range of $y = 14 - 5 \sin \left(\frac{\pi}{12}x - \frac{\pi}{6} \right)$, *express in interval notation.*

(b) (2 points) The period of $y = 2 - \cos(3\pi(\theta - 1))$.

(c) (2 points) The midline of $y = \frac{1}{4} \cos(6t) + \frac{9}{2}$.

10. A weight on a spring is released from a height of 10 cm, and oscillates sinusoidally between a height of 10 cm and 4 cm (as measured from a table). It takes 6 seconds for the weight to make a complete cycle.



- (a) (4 points) Use a sinusoidal function ($y = A \sin(B(t - h)) + k$ or $y = A \cos(B(t - h)) + k$) to model the height of the spring t seconds after it has been released.

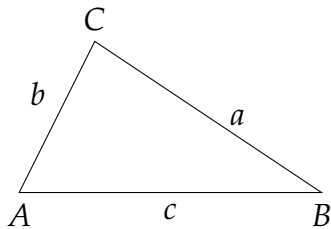
- (b) (4 points) Write an equation for the first time the weight is 7 cm above the table. Find a solution to this equation, giving your answer in terms of an inverse trig function and evaluate with correct units.

11. Find the reference angle for each θ and find the sine, cosine, and tangent value for each. (For example: $\cos(\text{original angle}) = \text{_____} \cos(\text{reference angle}) = \text{_____}$), leave exact answers. If the angle is given in radians then your answer should be in radians and vice versa.

(a) (2 points) $\theta = 240^\circ$

(b) (2 points) $\theta = \frac{3\pi}{4}$

12. (8 points) Given the triangle below:



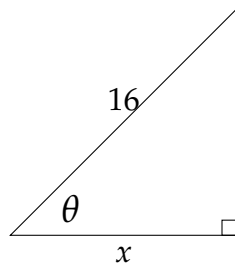
$A = 50^\circ$, $b = 9$, $c = 10$, find the exact value of length a .

13. (6 points) Determine if the identity

$$\left(\tan^2(x) + 1\right) \left(1 - \sin^2(x)\right) + 4 \csc(x) \sin(x) = 8$$

is true (that is, for any value of x). Show work to support your answer. It may be helpful to use trigonometric identities

14. With x and θ in the right triangle shown below and with $0 < \theta < \pi/4$, express the following in terms of x without using trigonometric functions



(a) (4 points) $\sin\left(\cos^{-1}\left(\frac{x}{16}\right)\right)$

(b) (4 points) $\sin(2\theta)$

15. (6 points) Find the inverse of $h(x) = \log(2x + 3)$.

16. Let's consider the functions $f(x) = x^2 + 2$ and $g(x) = \ln(3x)$.

(a) (4 points) Write an expression of $f(g(x))$. *Detail all the steps.*

(b) (2 points) What is the domain of $f(g(x))$? *Justify.*

End of exam