Math 131 Final Exam

Fall 2021 - December 15th, 2021

Name: ______________________________

Instructor Name: ______________________________

Did you have another exam 5:30-7:30 today (December 15). Circle: Yes No

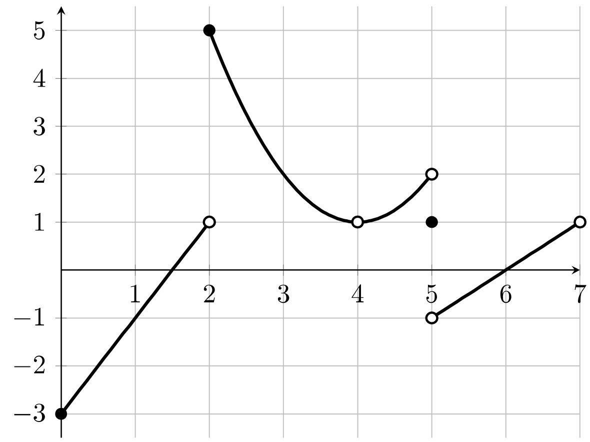
Page:	1	2	3	4	5	6	7	8	9	Total
Points:	20	16	20	12	9	25	22	12	16	152
Score:										

Answer the questions in the spaces provided on the question sheets. Show an appropriate amount of work (including appropriate explanation) for each problem, so that graders can see not only your answer but also how you obtained it. Include units in your answer when possible. You may receive 0 points for a problem where you show no work.

Instructions:

1. Do not open this exam until told to do so.
2. No books or notes may be used on the exam. There is an equation sheet on the last page of this exam.
3. Credit or partial credit will be given only when the appropriate explanation and/or algebra is shown.
4. Make sure your answer is clearly marked.
5. Read and follow directions carefully.
6. This exam has 14 questions, for a total of 152 points. There are 9 pages. Make sure you have them all.
7. You will have 120 minutes to complete the exam.
8. All cell phones and electronic devices (other than calculators) must be turned off during the exam.
9. Do not separate any of the pages of this exam except the last one containing the equation sheet. If they do become separated, write your name on every page and point this out to your instructor when you hand in the exam.
10. Calculators without internet access are allowed.
11. If you use graphs or tables to find an answer, be sure to include an explanation and sketch of the graph, and to write out the entries of the table that you use.

1. (12 points) Using the graph below, find each of the following. If the answer does not exist, write "DNE".



(a) $f(1)$

(c) $f(5)$

(e) $\lim_{x\to 4} f(x)$

(b) $f(4)$

(d) $\lim_{x\to 2} f(x)$

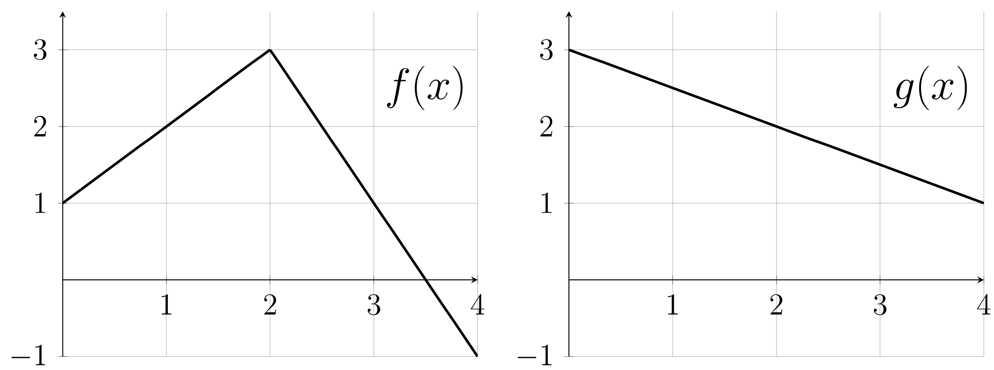
(f) $\lim_{x\to 5^-} f(x)$

2. Find each of the following limits exactly. If the answer does not exist, write "DNE". Use L'Hopital's Rule if it applies

(a) (4 points) $\lim_{x\to 2} \dfrac{x^3-7x^2+10x}{x^2+x-6}$

(b) (4 points) $\lim_{x\to\infty} \dfrac{x}{e^x}$

3. Use the graphs of $f(x)$ and $g(x)$ below to compute the derivatives. If the answer does not exist, write "DNE".



(a) (4 points) If $h(x) = f(x)g(x)$, compute $h'(1)$.

(b) (4 points) If $j(x) = f(g(x))$, compute $j'(1)$.

4. Find $\dfrac{dy}{dx}$ for the following; you do not have to simplify.

(a) (4 points) $y = x^2 \cdot 2^x - 3$

(b) (4 points) $y = \dfrac{e^x}{\cos(x)}$

5. For the function $f(t) = \dfrac{t}{1+t^2}$

(a) (10 points) Find critical points and perform the first derivative test to find local minima/maxima. *You only need to provide the t-value where the max/min occur.*

(b) (6 points) The second derivative is $f''(t) = \dfrac{2t^3 - 6t}{(t^2+1)^3}$. Find the inflection points for $f(t)$. Make sure to verify that they are inflection points. *You only need to provide the t-value where the inflection points occur.*

(c) (4 points) Find the global maximum and minimum for $f(t)$ on the closed interval $[0, 5]$.

6. (12 points) Provide a careful sketch of a graph of a single function $f(x)$ that satisfies the following six conditions. No formula is needed just carefully sketch and label your graph. Full credit will not be given for a graph that is not carefully labeled or that does not clearly satisfy the six conditions indicated.

i. $f(0) = 2$;

ii. $f(x)$ is continuous;

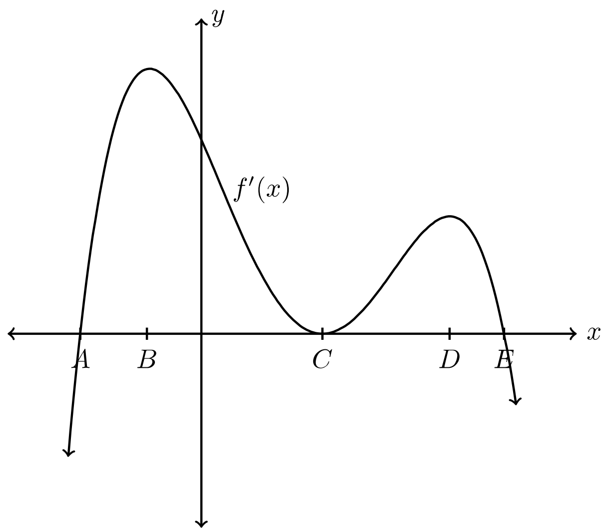
iii. $f'(x) < 0$ for $x < -4$ and for $x > 5$;

iv. $f'(x) > 0$ for $-4 < x < 5$;

v. $f''(x) > 0$ for $x < 0$;

vi. $f''(x) < 0$ for $0 < x$.

7. Consider the graph of the **DERIVATIVE** $f'(x)$ given below.



(a) (3 points) Identify all the critical points of the **function** $f(x)$.

(b) (3 points) Classify the critical points you found in part (a) as local minimum, local maximum, or neither.

(c) (3 points) Find all the inflection points of the **function** $f(x)$.

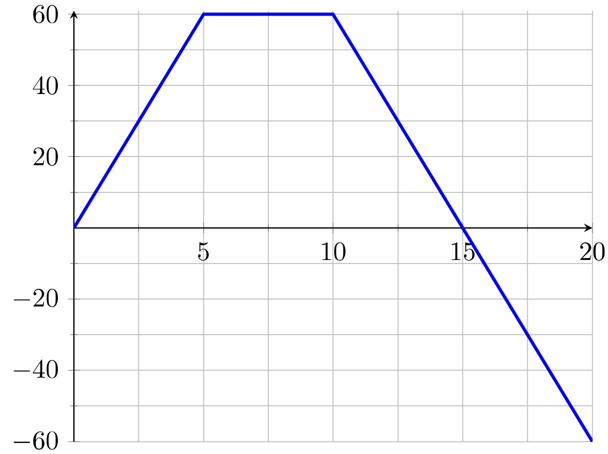
8. A manufacturer of baseball bats makes x bats at a cost of $C(x) = 4x + 10$ dollars. The revenue from the sale of x bats is given by $R(x) = 50x - 0.5x^2$ dollars.

   (a) (6 points) How many bats should be manufactured and sold to maximize profit?

   (b) (4 points) Use the second derivative to show that the number of bats you found gives a maximum profit.

9. (15 points) A grocery store wants to replace paper bags with a sturdy open top container, with a square base, that holds 4 ft^3 of groceries. What should the dimensions of the container be in order to use the least amount of material? Start by drawing a diagram.

10. This graph shows the velocity, in km/hr of a drone flying over a 24 hour period. Distance is measured in km away from its starting base. At time $t = 0$, the distance traveled is 0. Time is measured in hours and the velocity is measured in km per hour.

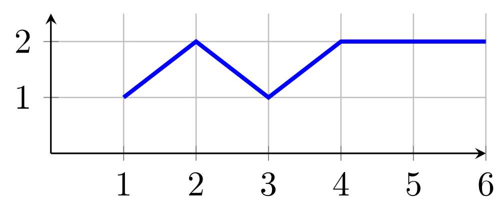


(a) (8 points) Complete the table for the **distance** the drone is from its starting base after t hours.

Time (hrs)	0	5	10	15	20
Distance (km)	0				

(b) (4 points) On what interval(s) is the **distance** from the base decreasing?

11. The graph of $f(x)$ is give below.



(a) (5 points) Use the figure to compute $\int_1^6 f(x)\,dx$.

(b) (5 points) What is the average value of f on $[1, 6]$?

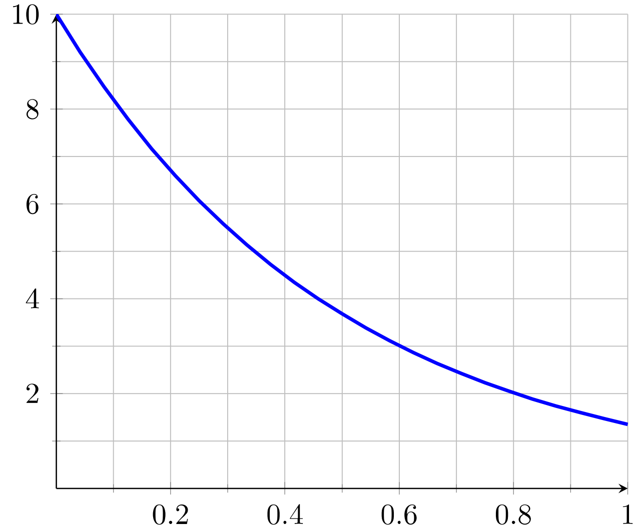
12. Find these indefinite integrals (don't forget the $+C$ where appropriate)

   (a) (4 points) $\displaystyle\int \left(x^4 + \frac{7}{x} + \frac{8}{x^3}\right)\,dx$

   (b) (4 points) $\displaystyle\int (7\cos(x) + 3e^x)\,dx$

   (c) (4 points) $\displaystyle\int 200(1.03)^x\,dx$

13. (a) (6 points) Use a left Riemann sum and $n = 5$ rectangles to approximate $\int_0^1 10e^{-2x}\,dx$. Draw the rectangles and show your work.



(b) (6 points) Evaluate $\int_0^1 10e^{-2x}\,dx$ exactly.

14. (4 points) The concentration of a medication in the plasma changes at a rate of $h(t)$ mg/ml per hour, t hours after the delivery of the drug. There is 250 mg/ml of the medication present at time $t = 0$ and $\int_0^3 h(t)\,dt = 480$. Determine the plasma concentration of the medication present three hours after the drug is administered. *Make sure to include units in your answer.*

Five derivative rules for operations on functions.

Constant Multiple Rule: $\frac{d}{dx}[cf(x)] = cf'(x)$

Sum and Difference Rule: $\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$

Product Rule: $\frac{d}{dx}[f(x) \cdot g(x)] = f'(x)g(x) + f(x)g'(x)$

Quotient Rule: $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$

Chain Rule: $\frac{d}{dx}[f(g(x))] = f'(g(x)) \cdot g'(x)$

Ten derivative rules for functions

Derivative of a Constant: $\frac{d}{dx}[c] = 0$, where c is a constant.

The Power Rule: $\frac{d}{dx}[x^n] = nx^{n-1}$

Exponential Functions: General Case: $\frac{d}{dx}[a^x] = a^x \cdot \ln(a)$

Exponential Functions: Special Case: $\frac{d}{dx}[e^x] = e^x$

Three Trigonometric Rules. $\frac{d}{dx}[\sin(x)] = \cos(x)$

$\frac{d}{dx}[\cos(x)] = -\sin(x)$

$\frac{d}{dx}[\tan(x)] = \sec^2(x) = \frac{1}{\cos^2(x)}$

Three Inverse Function Rules

$\frac{d}{dx}[\ln(x)] = \frac{1}{x}$

$\frac{d}{dx}[\arctan(x)] = \frac{1}{1+x^2}$

$\frac{d}{dx}[\arcsin(x)] = \frac{1}{\sqrt{1-x^2}}$

General Antiderivative Rules

If k is a constant $\int k\,dx = kx + C$

$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$, when $n \neq -1$

$\int a^x\,dx = \frac{a^x}{\ln(a)} + C$

$\int e^x\,dx = e^x + C$

$\int \cos(x)\,dx = \sin(x) + C$

$\int \sin(x)\,dx = -\cos(x) + C$

$\int \sec^2(x)\,dx = \tan(x) + C$

$\int \frac{1}{x}\,dx = \ln(|x|) + C$

$\int \frac{1}{1+x^2}\,dx = \arctan(x) + C$

$\int \frac{1}{\sqrt{1-x^2}}\,dx = \arcsin(x) + C$