Chapter 1

1. A square is inscribed in a circle. Express the area of the square as a function of the radius of the circle.

2. Eliminate the parameter to convert the parametric function $x = t + 1, \quad y = t^2 + t + 1$ into rectangular form and then graph it.

Chapter 2

3. Evaluate the following limit.
$$\lim_{x\to 0} \cos(x) + \sin(x)$$

4. Evaluate the limits or show that they do not exist. If you use any techniques, you must justify their use.

   (a) $\displaystyle\lim_{x\to 0} \frac{x^3}{x - \sin(x)}$

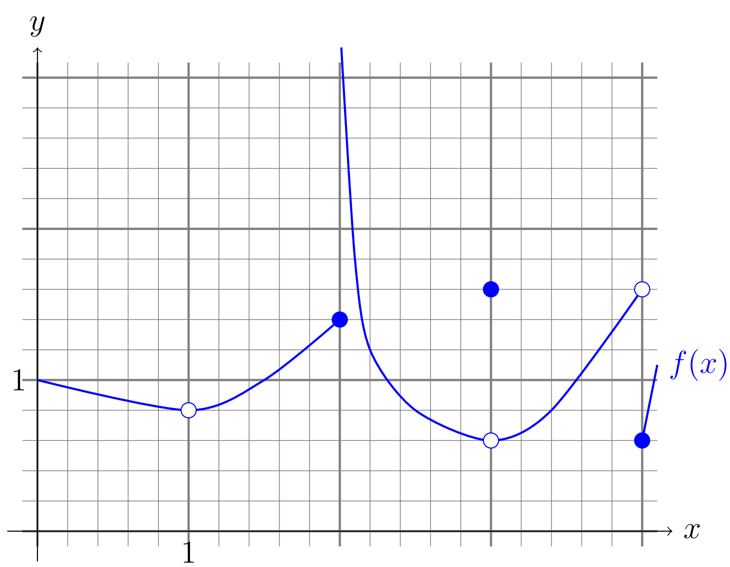
   (b) $\displaystyle\lim_{x\to 0} \frac{e^x - x^2/2 - x - 1}{x^5}$

5. Evaluate $\displaystyle\lim_{x\to 0} \frac{e^x(\cos(\pi x) - 1)}{x^4 \cos(\pi x)}$

6. Given that $f(x)$ is a function for which $\sin x \le f(x) \le \csc x$, for $0 \le x \le \pi$, find the limit of $f(x)$ as $x \to \pi/2$.

7. Find $\displaystyle\lim_{x\to 0} x^3 \cos(\tfrac{1}{x^2})$

8. Consider the function $f(x)$ whose graph is given below:



Fill in the values (or write DNE if they do not exist):

(a) $f(1) =$ ____________

(b) $f(2) =$ ____________

(c) $f(3) =$ ____________

(d) $f(3) =$ ____________

(e) $\lim_{x\to 1^-} f(x) =$ ____________

(f) $\lim_{x\to 2^-} f(x) =$ ____________

(g) $\lim_{x\to 3^-} f(x) =$ ____________

(h) $\lim_{x\to 4^-} f(x) =$ ____________

(i) $\lim_{x\to 1} f(x) =$ ____________

(j) $\lim_{x\to 2} f(x) =$ ____________

(k) $\lim_{x\to 3} f(x) =$ ____________

(l) $\lim_{x\to 4} f(x) =$ ____________

Chapter 3

9. The displacement (in meters) of an object moving in a straight line is given by $s = 1 + 2t + \frac{1}{4}t^2$, where is measured in seconds.

   (a) Find the average velocity over each time period.

      i. $[1, 3]$

      ii. $[1, 1.1]$

   (b) Find the instantaneous velocity when $t = 1$.

10. Consider the parametric equations $x(t) = \frac{1}{t^2+1}$ and $y(t) = t^3 + t$. Find $\frac{dx}{dt}$, $\frac{dy}{dt}$ and $\frac{dy}{dx}$.

11. Calculate $\dfrac{dy}{dx}$

   (a) $y = (x^4 - 3x^2 + 5)^3$

   (b) $y = \sqrt{x} + \dfrac{1}{\sqrt[3]{x^4}}$

   (c) $\dfrac{3x-2}{\sqrt{2x+1}}$

   (d) $y = \sin^2(\cos(\sqrt{\sin(\pi x)}))$

   (e) $\sin(xy) = x^2 - y$

(f) $x\tan(y) = y - 1$

(g) $y(x) = \ln\sqrt{\frac{x^2-4}{x^2+4}}$

12. Find an equation of the tangent to the curve, $y = 4\sin^2(x)$, at the given point $(\pi/6, 1)$.

13. Find all the points on the curve $x^2y^2 + xy = 2$ where the slope of the tangent line is 1.

$$\frac{d^2y}{dx^2} = \frac{d^2y}{du^2}\left(\frac{du}{dx}\right) + \frac{dy}{du}\frac{d^2u}{dx^2}$$

Chapter 4

14. Gravel is being dumped from a conveyor belt at a rate of $30\text{ft}^3/\text{min}$, and its coarseness is such that it forms a pile in the shape of a cone whose base diameter and height are always equal. (See Figure 1 for an illustration.) How fast is the height of the pile increasing when the pile is 10 ft high?

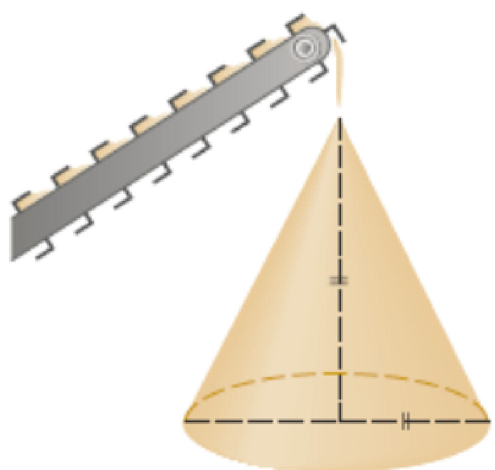


Figure 1: Gravel from a conveyer belt

15. Find the linear and quadratic polynomial approximations to the function $f(x) = xe^{2x}$ at $x = 1$.

16. Use a linear approximation to estimate $\sin(3)$ (the units here are radians).

17. A metal pole stands 5 feet tall in the center of a flat, paved lot. It is late afternoon and the sun casts the shadow of the pole onto the lot, and that shadow lengthens with time. Currently, the shadow is 12 feet long, and is increasing at a rate of 0.1 feet per minute. Find the rate of increase of the distance from the tip of the pole to the tip of the shadow.

18. Using the coordinate system (x,y) as (east, north), a ship sails from the dock 8 miles straight east. From then on, it sails straight north. When the ship is exactly 17 miles from its launching point, its distance from launch is increasing at a rate of 11 miles per hour. Find the speed of the boat.

19. For the function $f(x) = e^{\ln(x)+3}$ on the interval $[3, 10]$, find all critical points and all inflection points, as well as the maximum and minimum value.

20. Sketch a graph of the function $y = x^4 + 2x^3 - 9x^2 + 6$. Your graph should include all critical points and inflection points.

21. If

$$f(x) = \frac{x^2 - 4x + 3}{x^2 + 1},$$

   (a) Find the open intervals where f is increasing and where f is decreasing.

   (b) Find the open intervals where f is concave upward and where f is concave downward.

   (c) Find all extrema on I=[-10,10].

Chapter 5

22. Evaluate the following integrals.

   (a)

$$\int_0^2 (x^3 - 3x + 3)\ dx$$

   (b)

$$\int_1^9 \frac{2x^2 + x^2\sqrt{x} - 1}{x^2}\ dx$$

   (c)

$$\int \frac{-9x^2 + 10x}{\sqrt{3x^3 - 5x^2}}\ dx$$

   (d)

$$\int e^x \sin(e^x)\ dx$$

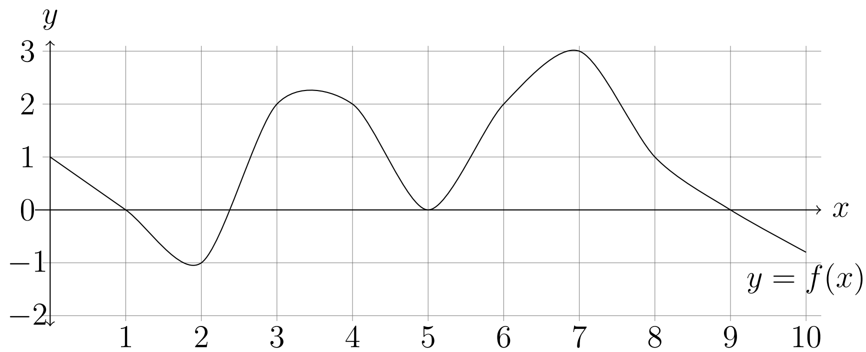
   (e)

$$\int_0^3 xe^{x^2}\ dx$$

(f)

$$\int_0^{\frac{\pi}{8}} \sec(2\theta)\tan(2\theta)\ d\theta$$

23. All parts of this problem refer to the following function:

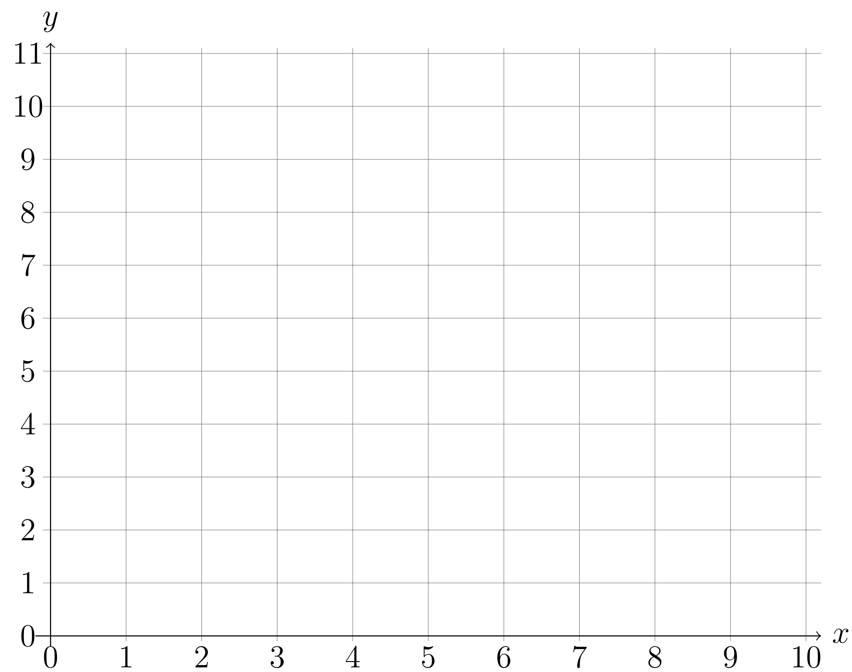


(a) Which is bigger: $\int_1^5 f(x)\,dx$ or $\int_4^8 f(x)\,dx$?

(b) Use a Reimann sum with left endpoints and $\Delta x = 2$ to estimate $\int_0^{10} f(x)\,dx$.

(c) If $F(x) = \int_1^x f(x)$, what is $F(3)$? What is $F'(3)$? What is $F''(3)$?

(d) Sketch $F(x)$ from part (c).



24. Use the fundamental theorem of calculus to evaluate $\frac{d}{dx}\int_9^{x^3} (e^t)(t^2 + 2t + 3)(\sin t)dt$

25. If $F(3) = 7$ and $\int_3^8 F'(x)dx = 15$ then what is $F(8)$?

26. Which function is an antiderivative of $f(x) = \frac{x^2-2x}{(x-1)^2}$?

(a) $\frac{x^3-x^2}{(x-1)^3}$

(b) $\frac{-x^2}{1-x}$

(c) $\frac{2}{(-1+x)^3}$