1. (6 points) Ten inches of snow is equivalent to one inch of rain. Write an equation for the amount of precipitation, measured in inches of rain, $r = f(s)$ as a function of the equivalent number of inches of snow $s$.

**Solution:**

$$r = f(s) = \frac{s}{10}$$

- 2 points | At least some expression giving $r$ as a function of $s$
- 1 point | Formula on the right track, may have $r = 10s$
- 3 points | Correct formula

2. (8 points) Given the function $f(x) = 2 - x^2$, compute the average rate of change of $f$ between $x = 1$ and $x = 4$. Show your work.

**Solution:**

$$\Delta f \over \Delta x = \frac{f(4) - f(1)}{4 - 1} = \frac{2 - 16 - (2 - 1)}{3} = \frac{-15}{3} = -5$$

- 2 points | Some quotient, might be $\Delta x/\Delta f$ or have incorrect endpoints
- 2 points | Correct quotient $\Delta f / \Delta x$
- 2 points | Progress toward numerical value, may have errors
- 2 points | Correct numerical value
3. We have $24 to spend on vegetables and fruit. A pound of vegetables costs $1 and a pound of fruit costs $2. The number of pounds of vegetables we can afford, $y$, is a function of the number of pounds of fruit we buy, $x$.

(a) (6 points) Find an equation relating $x$ and $y$.

Solution: $2x + y = 24$, or $y = -2x + 24$

- 1 point some expression relating $x$ and $y$
- 1 point $y$ is a linear function of $x$
- 3 points substantially correct, maybe a sign error
- 1 point correct equation

(b) (6 points) On the axes below:
- Graph your equation.
- Label each axis by writing the name of the variable and its units along the axis.
- Label the coordinates at the vertical and horizontal intercepts.
- Draw a dot at the point on your graph corresponding to a purchase of 2 pounds of fruit and label its coordinates.

Solution: 2 points correct graph
4 points 1 for each correct bullet point
Interpret “correct graph” as either correctly reflecting the practical description of the problem or correctly matching the expression the student gave in the previous part, to the student’s benefit.

4. (6 points) Write the equation of the line perpendicular to $4x + 3y = 9$ that passes through the point $(8, 5)$.

Solution: The slope of $4x + 3y = 9$ is $-\frac{4}{3}$, the perpendicular slope is $\frac{3}{4}$. The perpendicular line’s equation is $y = \frac{3}{4}x - 1$ or any equivalent equation.

- 1 point some linear function
- 1 points uses some slope derived from the given line
- 1 points correct slope
- 2 points substantially correct equation of line, may be some error
- 1 point all correct
5. Let \( f(x) = 2x - 7 \) and \( g(x) = \frac{2x - 3}{4x + 2} \).

(a) (3 points) Evaluate \( g(3) \).

Solution: \( g(3) = \frac{6 - 3}{12 + 2} = \frac{3}{14} \)

1 point | plug in 3 to \( g(x) \)
1 point | substantially correct evaluation
1 point | correct answer

(b) (3 points) Evaluate \( f(g(3)) \).

Solution: \( f(g(3)) = f\left(\frac{3}{14}\right) = 2 \times \frac{3}{14} - 7 = \frac{3}{7} - 7 = -\frac{46}{7} \approx -6.57 \)

1 points | use correct definition of composition
1 point | substantially correct evaluation
1 point | all correct

Decimal approximations are OK in evaluating function values.

(c) (4 points) Find all values of \( x \) solving \( g(x) = 3 \).

Solution:

\[
\begin{align*}
2x - 3 &= 3 \\
4x + 2 &= 3 \\
2x - 3 &= 12x + 6 \\
10x &= -9 \\
x &= -\frac{9}{10}
\end{align*}
\]

1 point | set \( g(x) = 1 \)
2 points | at least substantially correct algebra
1 point | correct algebra and solution
6. Use the graph of $f$ below to answer the following questions.

(a) (5 points) Fill in the blanks to give a piecewise-defined expression for $f$.

$$f(x) = \begin{cases} 
-x - 2 & \text{for } -5 < x < -3 \\
\frac{4}{4} & \text{for } -3 < x < 4 
\end{cases}$$

<table>
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<tr>
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<th>Description</th>
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<tr>
<td>2</td>
<td>non-constant linear part</td>
</tr>
<tr>
<td>1</td>
<td>constant linear part</td>
</tr>
<tr>
<td>2</td>
<td>correct subintervals for domain</td>
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(b) (3 points) Give the domain and range of $f$.

- Domain: $-5 < x < 4$
- Range: $1 < f(x) \leq 3$ and $f(x) = 4$

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<tbody>
<tr>
<td>1</td>
<td>correct domain</td>
</tr>
<tr>
<td>1</td>
<td>at least partially correct range, might be one interval only</td>
</tr>
<tr>
<td>1</td>
<td>correct correct range</td>
</tr>
</tbody>
</table>
7. The cost (in dollars) of producing \( x \) dryers is

\[ C = f(x) = 450 + 27x \]

(a) (5 points) Give an expression for \( f^{-1}(C) \).

**Solution:**

\[ f^{-1}(C) = \frac{C - 450}{27} \]

2 points | some evidence of solving \( y = C(x) \) for \( x \)
2 points | substantially correct expression, may have some mismatched variables
1 point | correct expression with matched variables

By “mismatched variables” I mean something like \( f^{-1}(C) = \frac{x - 450}{27} \)

(b) (3 points) Explain in a sentence the practical meaning of your expression, with correct units.

**Solution:** \( x = f^{-1}(C) \) is the number of dryers produced for a cost of \( C \) dollars.

8. (8 points) The graph of \( y = f(x) \) is given below.

![Graph showing intervals for increasing and concavity](image)

Give the intervals on which \( f \) is simultaneously …

(a) …increasing and concave up.

**Solution:** \( 6 < x < 8 \)

(b) …increasing and concave down.

**Solution:** \( 0 < x < 2 \)

(c) …decreasing and concave up.

**Solution:** \( 4 < x < 6 \)

(d) …decreasing and concave down.

**Solution:** \( 2 < x < 4 \)
Solution: 1 point | if given interval satisfies at least one criterion
1 point | both criteria satisfied

Any method of notating or describing the intervals is OK.
If you have had specific conversations with your classes you may choose otherwise, but I suggest not distinguishing between < and ≤.
9. (8 points) The graph of \( y = g(x) \) contains the point \((-6, 18)\). Find a point on the graph of each of the following transformations of \( g \).
   
   (a) \( y = -2g(x) \)
   
   (b) \( y = g(3x) \)
   
   (c) \( y = g \left( \frac{1}{2}(x - 2) \right) + 2 \)
   
   (d) \( y = \frac{1}{3}g(x) - 2 \)

   Solution: For each part:
   
   1 point | at least one correct coordinate
   1 points | both coordinates correct

10. A quadratic function passes through \((4, 5)\) and has a vertex at \((6, 2)\).

   (a) (5 points) Give a formula for the quadratic function. Write your formula giving \( y \) as a function of \( x \).

   Solution: \( y = \frac{4}{3}(x - 6)^2 + 2 \)

   1 point | \( y \) as some quadratic function of \( x \)
   1 point | evidence of using correct vertex form
   1 point | correctly using vertex coordinates
   1 point | attempt to solve for leading constant
   1 point | correct work

   (b) (3 points) Explain in a sentence what aspect of your formula tells you whether this is a concave up or concave down function.

   Solution: Since the leading constant is positive, the function is concave

   (3 points: This seems all-or-nothing.)
11. (8 points) The function $f$ is a rational function with a horizontal asymptote at $y = 0$. Its graph is shown below. Give a possible formula for $f(x)$.

Solution: $f(x) = \frac{x - 1}{(x - 2)(x - 5)}$

- 1 points: some rational function
- 2 points: correct numerator factor is present
- 1 point: no incorrect numerator factors
- 2 points: correct denominator factors present
- 2 points: no incorrect denominator factors

12. The concentration of a particular mineral in the water in a lake is proportional to the square of the depth. Let $S(x)$ be the mineral concentration at a depth of $x$ feet.

(a) (4 points) Write out the formula for $S(x)$ in terms of $x$ and the constant of proportionality $k$.

Solution: $S(x) = kx^2$

- 2 points: some correct proportionality relationship, may use $x$ instead of $x^2$
- 2 points: correct expression

(b) (4 points) At a depth of 10 feet, the mineral concentration is 20 grams per liter. Find $k$ and rewrite the formula for $S$ using it.

Solution: Solve $20 = 100k$ to get $k = 0.2$, so $S(x) = 0.2x^2$.

- 2 points: set up an equation to solve for $k$ based on the formula from part (a)
- 2 points: correctly solve for $k$
- 1 point: write out the correct formula for $S(x)$

(c) (2 points) At what depth is the mineral concentration 180 grams per liter?
Solution: Solve $180 = 0.0x^2$ to get $x = 30$.
- 1 point  
  set up $S(x) = 180$ using formula from (b)
- 1 point  
  solve for $x$ correctly