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Common final exam for MATH 131

Directions:

- This exam has 15 questions. Please check that your exam is complete.
- You have two hours to complete this exam. It will be graded out of 100 points.
- You must be in the final exam Zoom room designated for final exams (communicated to you by your instructor) for proctoring. Exams from students not in the Zoom proctor room will not be graded.
- Show your work. Answers (even correct ones) without the corresponding work will receive no credit.
- You may use a graphing calculator (including DESMOS) and your own notes from class. You may NOT use online software with built-in mathematical tools or libraries such as Mathematica or WileyPlus during the exam. The only computer use during the exam should be Zoom for proctoring, graphing calculator, and any writing tool you are using to transcribe your answers.
- You may not communicate with anyone besides the instructor during this exam.
- After you have finished your exam, convert it to a single pdf file and upload it to the test site in Sakai.

Good luck!

Final Exam

Question 1 (8 pts.)

Use the figure to find each of the following. Write $-\infty$ or ∞ if the limit is negative or positive infinity. Write DNE if the limit is undefined.

(a) (1 pt.) $\lim_{x\to 0} f(x)$

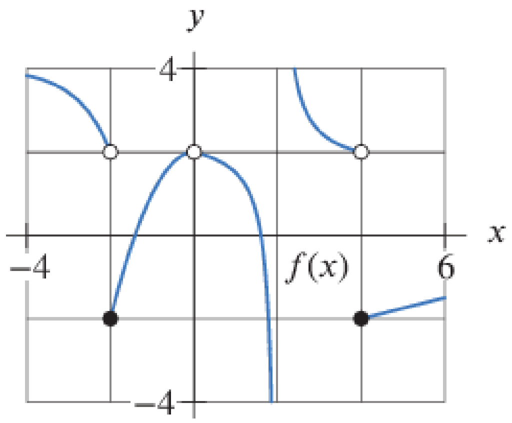
(b) (2 pts.) $\lim_{x\to 2^-} f(x)$

(c) (2 pts.) $\lim_{x\to 2^+} f(x)$

(d) (1 pt.) $\lim_{x\to 2} f(x)$

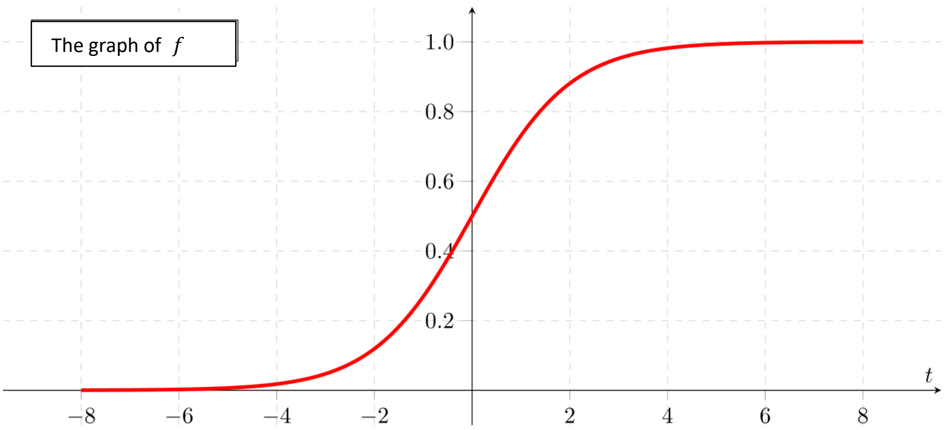
(e) (1 pt.) $\lim_{x\to -2^-} f(x)$

(f) (1 pt.) $\lim_{x\to -2^+} f(x)$



Question 2 (4 pts.)

The function f is graphed below.



Find the signs of the following numbers:

$$f(2), \qquad f'(2), \qquad f''(2)$$

$a)$ $+,+,+$ $\qquad$ $b)$ $+,+,-$ $\qquad$ $c)$ $+,-\ -$ $\qquad$ $d)$ $+,-,+$

$e)$ $-,+,+$ $\qquad$ $f)$ $-,+,-$ $\qquad$ $g)$ $-,-,+$ $\qquad$ $h)$ $-,-,-$

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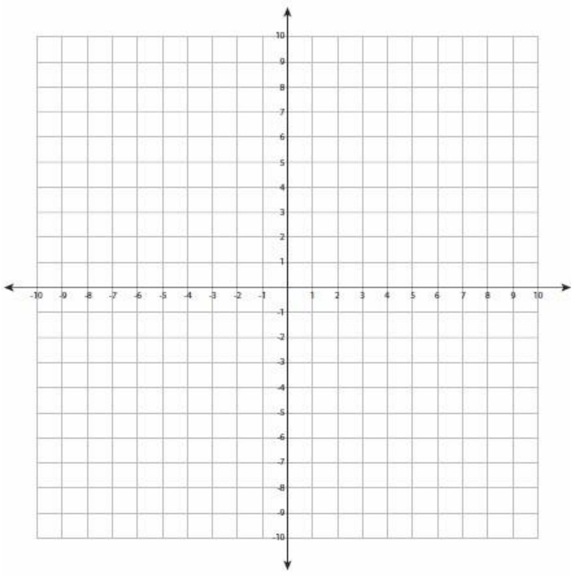
Question 3 (6 pts.)

Sketch the graph of a continuous function $y = f(x)$ that satisfies the following conditions:



1. $f'(x) > 0$ for $x < -2$,

2. $f'(x) < 0$ for $-2 < x < 5$,

3. $f'(x) = 0$ for $x > 5$,

4. $\lim_{x \to -2} f'(x)$ exists.

Question 4 (7 pts.)

The temperature, T, in degrees Fahrenheit, of a cold potato placed in a hot oven is given by $T = f(t)$, where t is the time in minutes since the potato was put in the oven.

(a) (2 pts.) What is the sign of $f'(t)$? Explain.

(b) (2 pt.) What are the units of $f'(25)$?

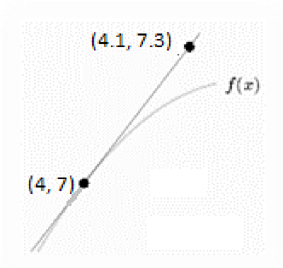
(c) (3 pts.) What is the practical meaning of the statement $f'(25) = 2$?

Question 5 (7 pts.)

Find the equation of the tangent line to the graph of $f(x) = \frac{2x-6}{ax+1}$ at the point at which $x = 0$. (a is a non-zero constant).

Question 6 (7 pts.)

Use the figure below to calculate the derivative, $h'(4)$ if $h(x) = (f(x))^4$.



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Question 7 (8 pts.)

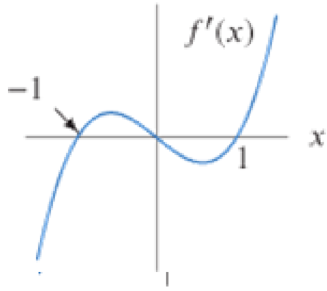
The function f is defined for all x. The graph shows f', the derivative of f. Use the graph to decide:

a. (2 pts.) Over what interval(s) is f increasing? Explain.

b. (2 pt.) List the x-coordinates of all local minima of f. If none, write DNE.

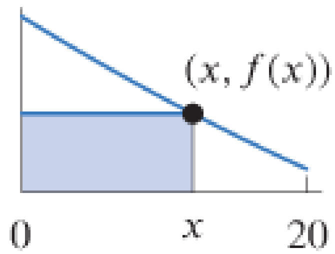
c. (2 pts.) Over what intervals is f concave up? Explain.

d. (2 pts.) List the x-coordinates of all inflection points of f. If none, write DNE.



Question 8 (9 pts.)

Find the x-value maximizing the shaded area. One vertex is on the graph of $f(x) = \frac{x^2}{3} - 40x + 800$, where $0 \le x \le 20$. Explain why your x-value maximizes the area.



Question 9 (6 pts.)

Find the constants $b, c,$ and d in a cubic polynomial $f(x) = x^3 + bx^2 + cx + d$ so that the polynomial has a critical point at $x = 5$ and an inflection point at $(7, 2)$. Show your work.

Question 10 (5 pts.)

A small tie shop sells ties for $\$3.50$ each. The daily cost function is estimated to be $C(x)$ dollars, where x is the number of ties sold on a typical day and

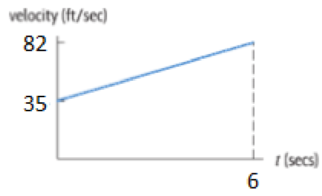
$$C(x) = 0.0006x^3 - 0.03x^2 + 2x + 20$$

Find the value of x that will maximize the store's daily profit. Assume you sell everything you produce. Provide an exact answer.

Question 11 (4 pts.)

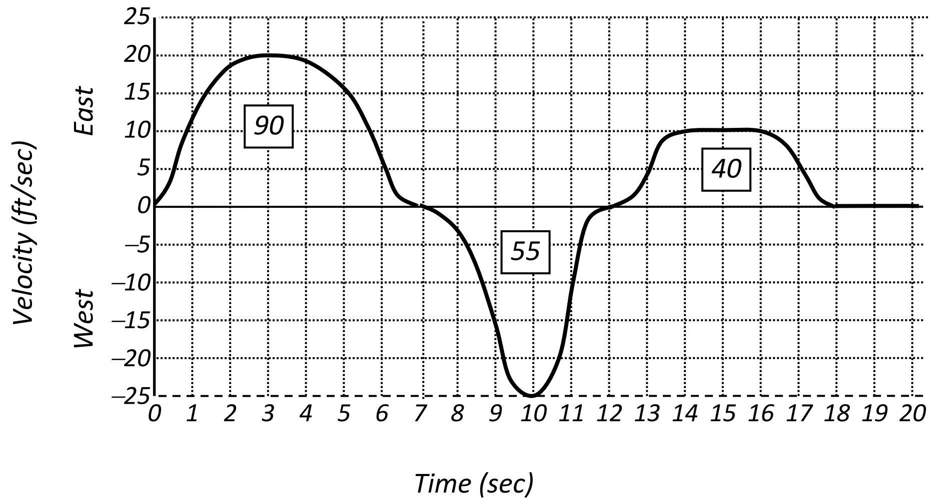
A car accelerates at a constant rate from 35 ft/sec to 82 ft/sec in 6 seconds as shown in the figure below.

How far does the car travel while it is accelerating? Show your work.



Question 12 (10 pts.)

Mr. Rabbit lives in a rabbit hole in the middle of a forest path. The path runs East & West. One morning, Mr. Badger made a graph of Mr. Rabbit's velocity as he ran along the path. The numbers in the squares are the approximate area of the regions above and below the axis.



(a) (2pts.) At what time(s) did Mr. Rabbit change direction? Explain.
(b) (2 pts.) When is Mr. Rabbit going the fastest? Explain.
(c) (2 pts.) At what speed is Mr. Rabbit going the fastest?
(d) (2 pts.) When is the rabbit furthest to the East?
(e) (2pts.) What is the total distance traveled by Mr. Rabbit? Show your work.

Question 13 (4 pts.)

The concentration of a medication in the plasma changes at a rate of $h(t)$ mg/ml per hour, t hours after the delivery of the drug. There is 220 mg/ml of the medication present at time $t = 0$, $\int_1^2 h(t)\,dt = 160$ mg/ml, $\int_2^3 h(t)\,dt = 110$ mg/ml, and $\int_0^3 h(t)\,dt = 375$ mg/ml.

a) (2 pts.) Write a sentence explaining the meaning of $\int_0^3 h(t)\,dt$ in the context of the problem.
b) (2 pts.) What is the concentration of the medication in the plasma present three hours after the drug is administered? Show your reasoning and use units in your answer.

Question 14 (6 pts.)

Evaluate the following expressions using the table. Give exact values if possible; otherwise, make the best possible estimates using left-hand Riemann sums.

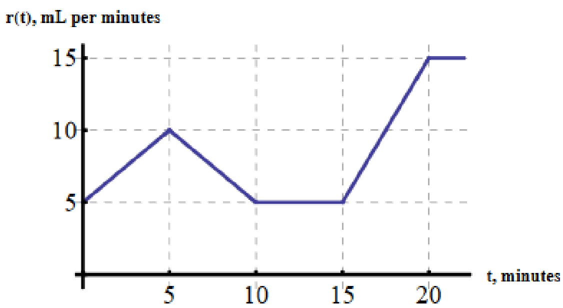
t	0	1	2	3	4	5
f(t)	1	3	5	6	8	9

a) (3 pts.) $\int_0^5 f(t)\, dt$

b) (3 pts.) $\int_2^5 f'(t)\, dt$

Question 15 (9 pts.)

There is a leaky faucet in the student's kitchen sink. The leak drips water from the faucet into the sink at a constant rate of 10mL per minute. The water is also draining out of the sink, but the rate the water drains out is not constant because of the pile of dirty dishes present. Let $r(t)$ be the rate, in mL per minute, that the water is draining out of the sink t minutes after 7:00pm. The graph of $r(t)$ can be found below. Assume the sink has 5mL of water in it at 7:00.



(a) (3 pts.) How much water drains out of the sink between 7:00 and 7:15? Show how you arrived at your answer.

(b) (3 pts.) How much water drips into the sink between 7:00 and 7:15? Show how you arrived at your answer.

(c) (3 pts.) Suppose that at 7:20, the student removes the dirty dishes from the sink and water is allowed to drain freely at a rate of 15mL per minute. How long will it take for the sink to have no water in it? (Show your work.)

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