Final Exam Study Guide

1. Use the graph of the function *f*(*x*), to find the following values

(a) $\lim_{x \to 0} f(x) =$

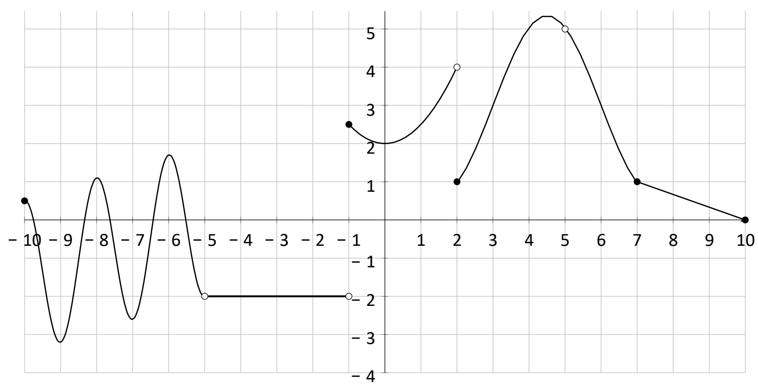
(b) $\lim_{x \to -1^+} f(x) =$

(c) $\lim_{x \to -1^-} f(x) =$

(d) $\lim_{x \to 2} f(x) =$

(e) $\lim_{x \to 5} f(x) =$

(f) Identify the intervals where $f(x)$ is continuous.



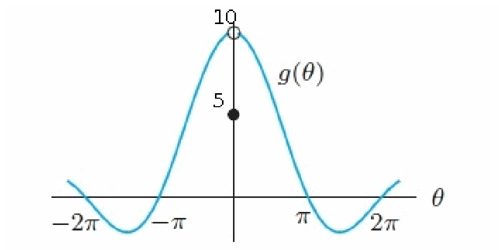
2. Find a value of k, if any, making $h(x)$ continuous on $[0, \infty)$.

$$h(x) = \begin{cases} e^{kx}, & 0 \le x < 2 \\ x, & x \ge 2 \end{cases}$$

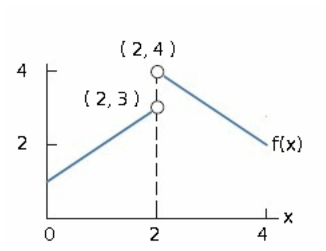
3. Is the following function continuous on $[-1,1]$?

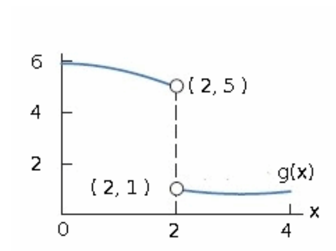
$$f(x) = \begin{cases} \dfrac{x}{|x|}, & x \ne 0 \\ 0, & x = 0 \end{cases}$$

4. Discuss the continuity of the function g graphed in the following figure:



5. Estimate the limits using the following graphs.





If the limit does not exist, enter DNE.

$$\lim_{x \to 2^-} [f(x) + g(x)]$$

$$\lim_{x \to 2^+} \frac{f(x) - 3x}{g(x)}$$

6. Find the following limits for $f(x) = 4x^7 + 25x^6 - 35x^5 - 100x^4 + 9x + 12$.

$$\lim_{x \to -\infty} f(x)$$

$$\lim_{x \to \infty} f(x)$$

7. Find the following limits for $f(x) = \frac{5x^3+13x^2+46}{9x^3+6x+11}$.

$$\lim_{x \to -\infty} f(x)$$

$$\lim_{x \to \infty} f(x)$$

8. Evaluate the following limit.

$$\lim_{x \to \infty} \frac{x - 6}{6 + 7x^2}$$

9. Evaluate the following limit.

$$\lim_{x \to -\infty} \frac{x^2 + 7}{x + 4}$$

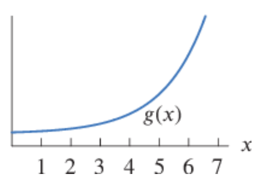
10. Evaluate the following limit. Show the computation steps.

$$\lim_{x \to \infty} \frac{9^{-x} + 4}{2^{-x} + 5}$$

11. The table below shows values of $f(x) = x^4$ near $x = 2$ (to three decimal places). Use it to estimate $f'(2)$.

x	1.998	1.999	2.000	2.001	2.002
x^4	15.936	15.968	16.000	16.032	16.064

12. Consider the function $f(x) = -x^2 + ax$, where a is a non-zero constant. What is the slope of the tangent line to the graph of at the non-zero x-intercept of the graph? Show all justificative work.

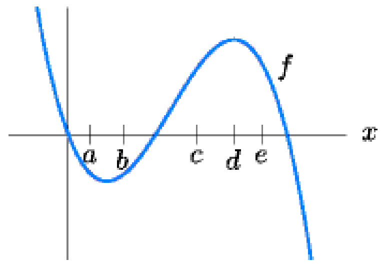


13. The graph of $y = g(x)$ is shown in above. Which is larger in each of the following pairs?

(a) $g(1)$ or $g(4)$? Justify.

(b) $g'(1)$ or $g'(3)$? Justify.

14. The figure below shows the graph of f. Match the derivatives in the table with the points $a, b, c, d,$ and e.



x	$f'(x)$
	0
	0.5
	2
	-0.5
	-2

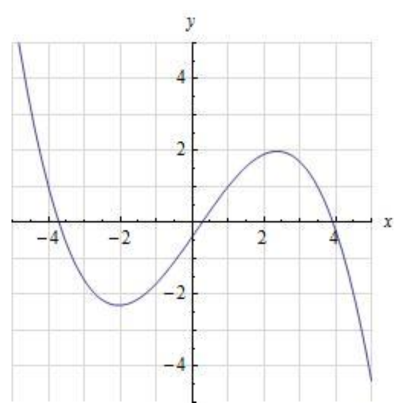
15. Use algebra to evaluate the limit.

$$\lim_{h\to 0}\frac{(-10+h)^2-100}{h}$$

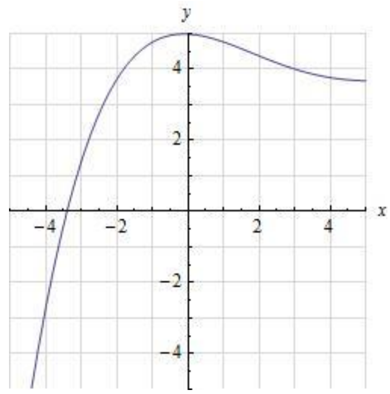
16. Find the derivative of $g(t) = 4t^2 + 2t$ at $t = -1$ by using the difference quotient.

17. Find the equation of the line tangent to the function, $f(x) = 7x^2$ at the point $x = 2a$.

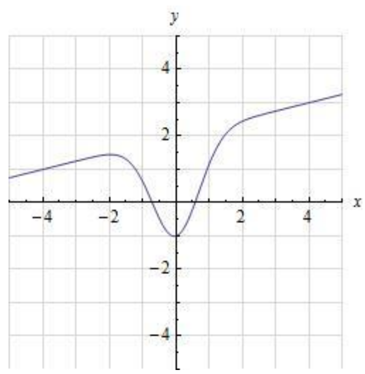
18. Sketch the graph of the derivative of the function $f(x)$ given below.



19. Sketch the graph of the derivative of the function $f(x)$ given below.



20. Sketch the graph of the first and second derivative of the function $f(x)$ given below.



21. The time for a chemical reaction, T (in minutes), is a function of the amount of catalyst present, a (in milliliters), so $T = f(a)$.

(a) If $f(7) = 17$, what are the units of 7? What are the units of 17? What does this statement tell us about the reaction?

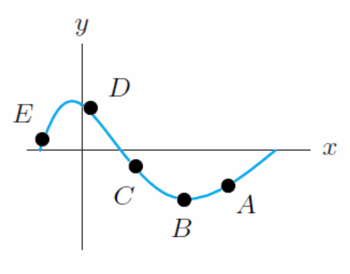
(b) If $f'(7) = -5$, what are the units of 7? What are the units of -5? What does this statement tell us?

22. The cost, C (in dollars), to produce q quarts of ice cream is $C = f(q)$. In each of the following statements, what are the units of the two numbers? In words, what does each statement tell us?

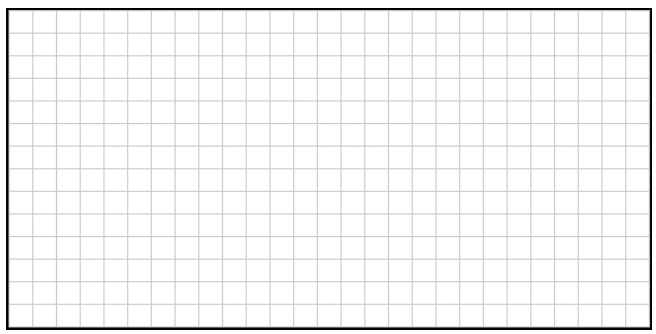
(a) $f(200) = 700$.

(b) $f'(200) = 5$

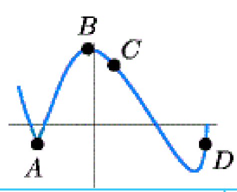
23. At one of the labeled points on the graph both $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ are positive. Find the point and justify your choice.



24. Sketch the graph of a function whose first derivative is everywhere negative and whose second derivative alternates over four regions between being positive and negative. In the first region, the second derivative is positive.

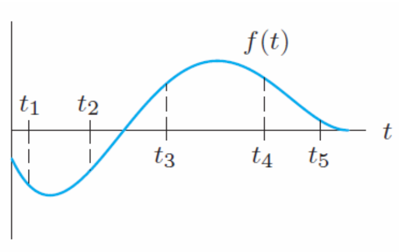


25. At exactly two of the labeled points in the figure below, the derivative f' is 0; the second derivative f'' is not zero at any of the labeled points. Give the signs of f, f', f'' at each marked point.



Point	f	f'	f''
A			
B			
C			
D			

26. The following graph gives the position, $f(t)$, of a particle at time t. Select each of the marked values of t for which the statements could be true.



(a) The position is positive.

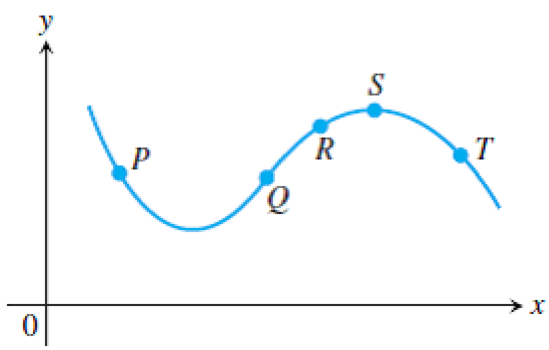
(b) The velocity is positive.

(c) The acceleration is positive.

(d) The position is decreasing.

(e) The velocity is decreasing.

27. The graph of f' (not f) is given in the figure below.



(a) At which of the marked values of x is $f(x)$ greatest?

(b) At which of the marked values of x is $f(x)$ least?

(c) At which of the marked values of x is $f'(x)$ greatest?

(d) At which of the marked values of x is $f'(x)$ least?

(e) At which of the marked values of x is $f''(x)$ is greatest?

(f) At which of the marked values of x is $f''(x)$ is least?

28. Find the derivative of the function given below. Assume that a, b, c, k are constants.

$$f(x) = \frac{cx + k^2}{ax^2 - b}$$

29. The differentiable functions f and g have the values shown below.

$$f(2) = 3 \text{ and } f'(2) = 5$$

$$g(2) = 4 \text{ and } g'(2) = -2.$$

30. For each of the following functions h, find $h'(2)$.

(a) $h(x) = f(x) + g(x)$

(b) $h(x) = f(x)g(x)$

(c) $h(x) = \frac{f(x)}{g(x)}$

31. Use the figure below and the chain rule to estimate the derivative. The graph of $f(x)$ has a sharp corner at $x = 2$.

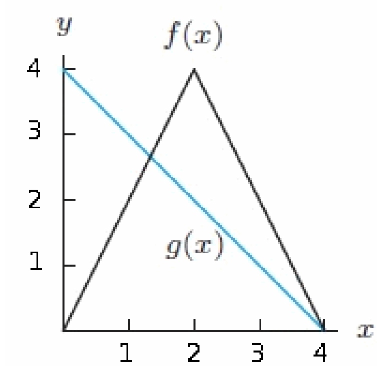
Let $w(x) = g(g(x))$ and $v(x) = f\big(g(x)\big)$.

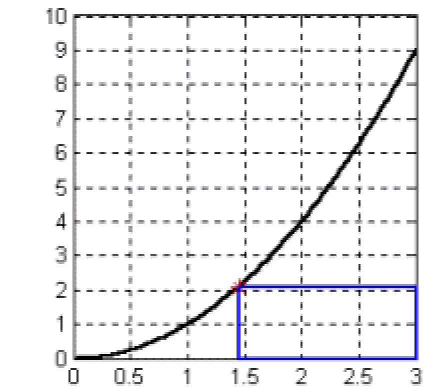
If the derivative does not exist, enter DNE.

(a) Find $w'(x)$ and $v'(x)$.

(b) Find $w'(1)$.

(c) Find $v'(1)$.





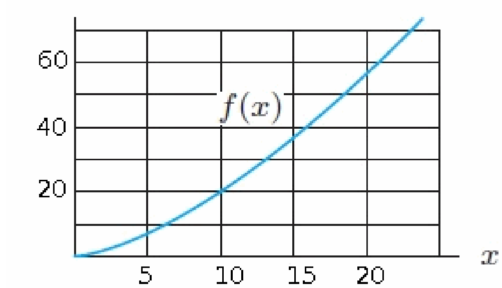
32. The parabola $y = x^2$ is sketched over the interval $[0, 3]$. A rectangle is inscribed under the curve as shown in the picture. Determine the inscribed rectangle of maximum area.

33. The total cost (in dollars) of producing q goods is given below:

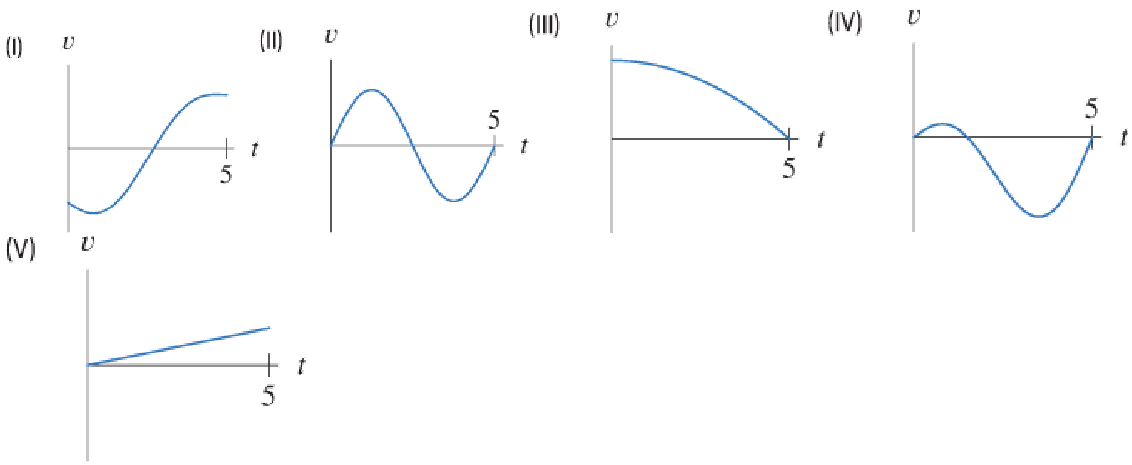
$$C(q) = 3 + 0.5q + 0.005q^3$$

Assuming that each item is sold for \$15.5 and everything that is produced is sold, find the quantity q that maximizes the profit and the maximum profit.

34. Using the figure, where $f'(5) = 2.1, f'(10) = 3,$ $f'(15) = 3.7, f'(20) = 4.2$, find $(f^{-1})'(20)$.



35. The graphs in the figure below represent the velocity, v, of a particle moving along the x-axis for time $0 \leq t \leq 5$. The vertical scales of all graphs are the same. Identify the graph showing which particle:



(a) Has a constant acceleration.

(b) Ends up farthest to the left of where it started.

(c) Ends up the farthest from its starting point.

(d) Experiences the greatest initial acceleration.

(e) Has the greatest average velocity.

(f) Has the greatest average acceleration.

36. Find the constants α, β and γ in a cubic polynomial, $f(x) = -2x^3 + 3\alpha x^2 - 6\beta x - 5\gamma$, that has a critical point at $x = 1$, an inflection point at $(-1, 0)$.

37. Evaluate the expressions using the table. Give exact values if possible; otherwise, make the best possible estimates using left-hand Riemann sums.

t	0	0.2	0.4	0.6	0.8	1
f(t)	0.8	0.1	0.5	0.2	0.7	0.3
g(t)	0.8	3	2.7	4.5	8.9	0.5

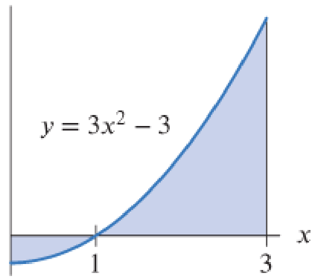
a) $\int_0^1 f(t)\,dt$

b) $\int_{0.2}^{0.8} g'(t)\,dt$

38. a) Given that $\int_0^4 f(x)dx = -1$, $\int_2^4 f(x)dx = 9$, evaluate $\int_2^0 f(x)dx$.

b) Given that $\int_8^{12}(5f(x) - 7)dx = 13$, find $\int_8^{12} f(x)dx$.

39. Find the exact area of the shaded region in the figure below between $y = 3x^2 - 3$ and the x-axis.



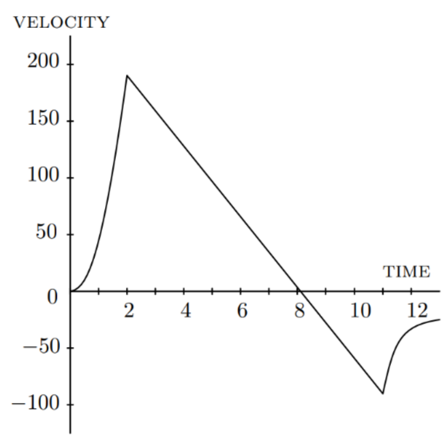
40. The figure below shows the graph of the velocity of a model rocket for the first 12 seconds after

launch.

a) Assuming the rocket was launched from ground level, about how high did it go?

b) Assuming the rocket was launched from ground level, about how high was the rocket 12 seconds after launch?

c) What is the rocket's acceleration at $t = 6$ seconds? At $t = 2$ seconds?



41. Using the graph of f in the figure below, arrange the following quantities in increasing order, from least to greatest.
Circle the correct order.

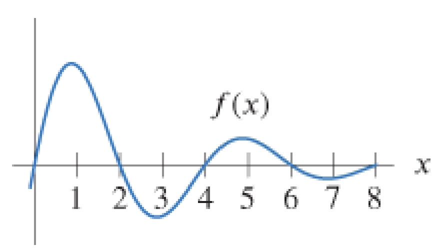
(i) $\int_0^2 f(x)\,dx < \int_2^4 f(x)\,dx < \int_4^6 f(x)\,dx$

(ii) $\int_4^6 f(x)\,dx < \int_4^2 f(x)\,dx < \int_1^2 f(x)\,dx$

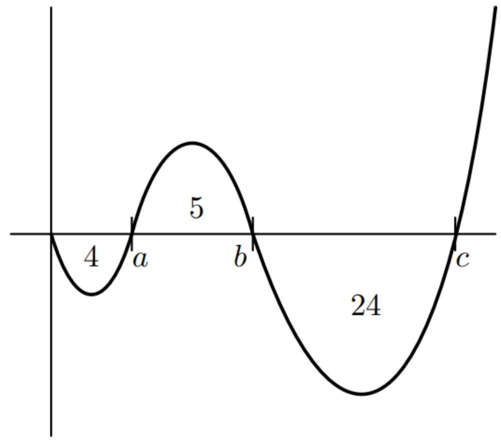
(iii) $\int_1^2 f(x)\,dx < \int_0^2 f(x)\,dx < \int_4^8 f(x)\,dx$

(iv) $\int_2^6 f(x)\,dx < 0 < \int_6^8 f(x)\,dx$

(v) None of the above



42. A particle moves along the x-axis. Its initial position at $t = 0$ sec is $x(0) = 15$. The graph on the right shows the particle's velocity $v(t)$. The numbers are areas of the enclosed figures.

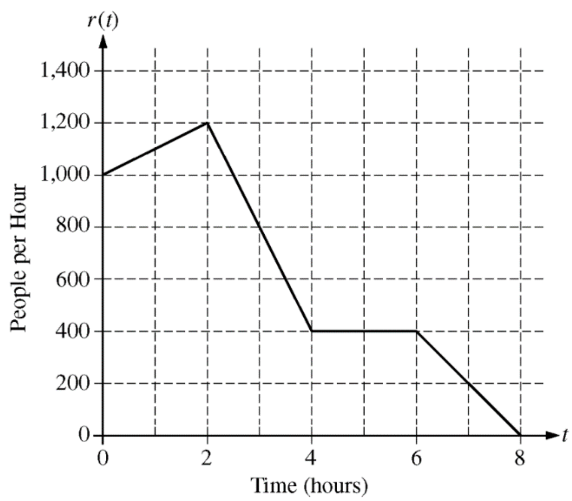


a) What is the particle's displacement between $t = 0$ and $t = c$?

b) What is the total distance traveled by the particle in the same time period?

c) Give the positions of the particle at times a, b, and c.

d) Approximately where does the particle achieve its greatest positive acceleration on the interval $[0, b]$? On $[0, c]$?



43. There are 700 people in line for a popular amusement-park ride when the ride begins operation in the morning. Once it begins operation, the ride accepts passengers until the park closes 8 hours later. While there is a line, people move onto the ride at a rate of 800 people per hour. The graph above shows the rate, $r(t)$ at which people arrive at the ride throughout the day. Time t is measured in hours from the time the ride begins operation.

(a) How many people arrive at the ride between $t = 0$ and $t = 3$? Show the computations that lead to your answer.

(b) Is the number of people waiting in line to get on the ride increasing or decreasing between $t = 2$ and $t = 3$? Justify your answer.

(c) At what time t is the line for the ride the longest? How many people are in line at that time? Justify your answers.

(d) Write, but do not solve, an equation involving an integral expression of r whose solution gives the earliest time t at which there is no longer a line for the ride.