

MATH 117 FINAL EXAMINATION

MAY 2012

Time Limit: 3 hours

Instructions: Answer any 18 of the 22 questions. You may answer more than 18 to obtain extra credit. In each problem, be certain to justify your answer! You will receive little or no credit for simply writing a correct answer.

1. Max and Albertine move to Las Vegas on January 1st, 2013. Max arrives with \$13,000 and loses 15% each year gambling. Albertine arrives with \$2,630 and promptly opens a savings account that pays 3.7% annual interest compounded monthly.

(a) When will Max and Albertine have the same amount of money?

(b) When will Albertine have twice as much money as Max?

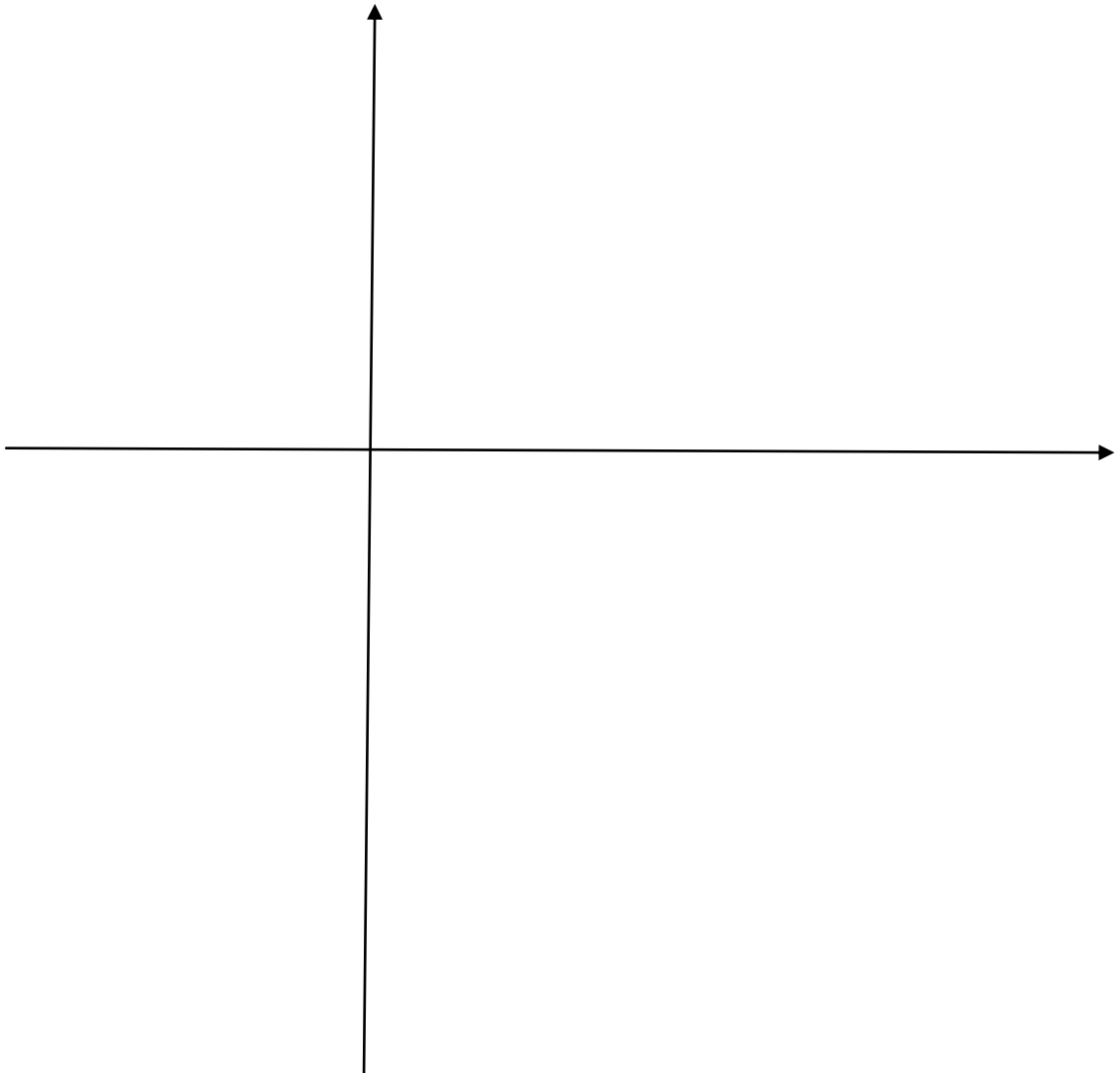
2. (a) Solve for t : $\log_3(3t + 5) - \log_3(t - 1) = 3$

(b) If $\log_b a = c$, find $\log_{b^3}(a^4)$ in terms of c .

(c) Evaluate, *without the use of a calculator*: $123456^2 - 123454^2$. *To receive and credit, you must explain how you obtained your answer.*

3. Sketch the following rational function. Be certain to include any (and all) zeroes and singularities. Perform a sign analysis and analyze limiting behavior. List all horizontal and vertical asymptotes.

$$y = \frac{(x-3)^3(x-4)^2}{(x^2+x+5)(x-1)(x-2)^2}$$



4. By using four rectangles, show that the area under the curve $y = 1/x^2$ between $x = 1$ and $x = 3$ must lie between 0.48 and 0.93. Sketch!

5. Let $f(x) = \frac{x}{x+2}$. Compute and *simplify fully* $\frac{f(x+h) - f(x)}{h}$.

6. Find a *square root* of $45 - 28i$. Carefully show all of your work! (*No credit will be given for a correct guess.*)

7. The population of Betaville triples every 14 years. On January 1, 2012, there were 109 inhabitants of Betaville.

(a) Find the population of Betaville on January 1, 2043.

(b) When will the population of Betaville equal 1,000,000?

8. (a) Solve for x .

$$\log_4 x + 1 = \frac{15}{4\log_4 x}$$

(b) Solve for t : $5 = 31 - 2\{1 + 3(2 - (3 - \log_2 t))\}$

9. Suppose that Isaac's nephew, Jacques, is born today. Isaac would like to give Jacques a check for 13,000 euros when Jacques turns 16 years old.

(a) How much should Isaac deposit today at the Second Third Loyola Bank given that the bank pays annual interest of 3.93% compounded *weekly*?

(b) How much should Isaac deposit today at the Eighth Fourth Loyola Bank given that the bank pays annual interest of 4.2% compounded *continuously*?

10. Suppose that a radioactive isotope loses 19.5 % of its initial amount in 407 years.

Find the half-life of the isotope.

11. An estate was divided among three persons, Albertine, Boris and Carlos, in such a way that Albertine's share is four times that of Boris, and the share of Boris twice that of

Carlos. Albertine received \$1700 more than Carlos. How much did each receive? *You must show your work to obtain credit.*

12. Let $H(x) = \sqrt{1 + 3e^x}$.

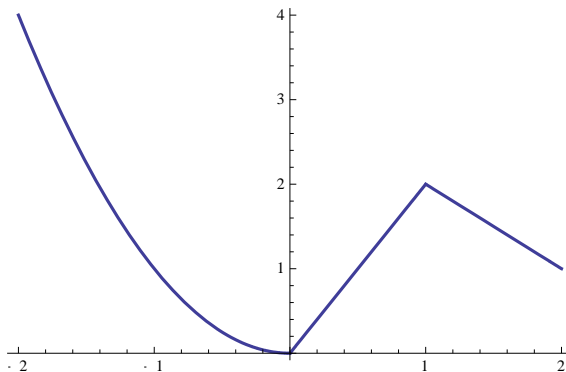
(a) What is the *domain* of H ?

(b) What is the *range* of H ?

(c) Find functions, F and G , each simpler than H , such that $H = F \circ G$.

(d) Find a formula for the *inverse* of H .

13. Let $F(x)$ be a function with domain $[-2, 2]$ and range $[0, 4]$. The graph of F is displayed below:



(a) Let $G(x) = 1 + 2F(x+3)$.

What is the *domain* of G ?

What is the *range* of G ?

Sketch the graph of G .

(b) Let $H(x) = -F(1 - 2x)$.

What is the *domain* of H ?

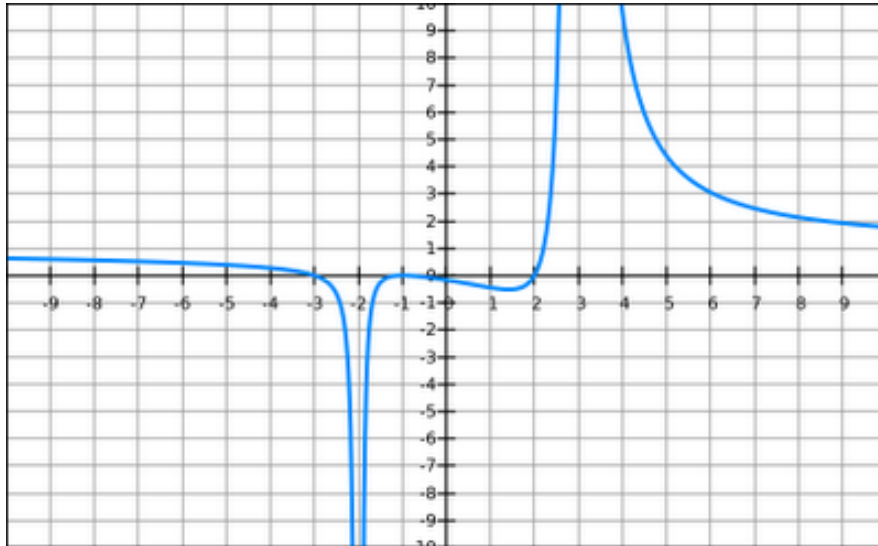
What is the *range* of H ?

Sketch the graph of H .

14. Express the following improper rational function as a polynomial plus a proper rational function:

$$\frac{x^6 + 2x^5 + x^4 - x^2 + 2x - 1}{x^2 - x + 3}$$

15. Find an *equation* of a rational function whose graph is given below



Assume:

- (1) $y \rightarrow 1$ as $x \rightarrow \pm \infty$
- (2) the vertical asymptotes are $x = -2$, $x = 3$
- (3) zeroes at $x = -3$, $x = -1$, $x = 2$

16. (a) Write an *equation of the circle* with center $C = (1, 0)$ and radius of 13.

(b) Consider the line $3x + 2y = 42$. Find *all points of intersection* of the line and the circle. Show your work! *Sketch*.

17. (a) Find the *domain* of the function $G(x) = x \ln(x + 8) + e^x \ln(9 - 3x) + 2012$

(b) How many digits does 17^{4321} have?

18. Find *all roots* of the polynomial: $p(x) = 2x^3 - 11x^2 - x + 30$

Hint: $x = 2$ is a root of this polynomial.

19. Using the method of *completing the square*, determine the maximum or minimum value *achieved* by each of the following:

(a) $f(x) = 1 + 4x + 2x^2$

(b) $g(x) = -x^2 + 6x - 19$

20. *Simplify fully* the following complex fraction:

$$\frac{9x^2 - 64}{x - 1 - \frac{1}{1 - \frac{x}{x + 4}}}$$

21. Solve the following equation for x . Express your answer to the nearest hundredth.

$$2012 (0.91)^{8x+1} = 5^{x-9} e^{0.1x}$$

22. Solve for x :

$$(x + 1)(x + 2)(x + 6) = x^3 + 9x^2 + 4(7x - 1)$$

