

Math 117 Final Examination

AUGUST 8, 2012

Name _____

Instructions. This examination contains 35 problems two of which are *extra credit* problems. *Please show your work where appropriate.*

1. True or false? Write 'T' for true and 'F' for false.

(a) _____ $(a + b + c)^2 = a^2 + b^2 + c^2$

(b) _____ $\sqrt[4]{x + y} = \sqrt[4]{x} + \sqrt[4]{y}$

(c) _____ $\log_b a \cdot \log_a b = 1$

(d) _____ $0^0 = 1$

(e) _____ $\sqrt{\frac{u}{v}} = \frac{\sqrt{u}}{\sqrt{v}}$

(f) _____ $\left(\frac{81}{10000}\right)^{\frac{3}{2}} = \frac{729}{1000000}$

(g) _____ $(\ln 7)^x = x \ln 7$

(h) _____ $\log_a 37 > \log_b 37$ if $a < b$

(i) _____ $(\log_{10} x)(\log_{10} y) = \log_{10} xy$

2. Find the *distance* between $(7, 11)$ and $(17, 13)$. *Give an exact answer and simplify completely.*

Distance = _____

3. Find the *midpoint* between the points in problem 2.

Midpoint: _____

4. Find the *equation* of a line that has x -intercept $(11, 0)$ and is perpendicular to the line that passes through the points $(5, 4)$ and $(7, -1)$.

Equation: _____

5. True or false? Write 'T' for true and 'F' for false.
- (a) _____ If the slope of a line is close to 1, the line perpendicular to it has an undefined slope.
- (b) _____ $x = 5$ is a vertical line that passes through the point $(5, -2012)$.
- (c) _____ A vertical line has slope 0.
- (d) _____ The lines $y = x$ and $9x - 9y = 729$ are parallel lines.
- (e) _____ A line always represents a function.
6. *Complete the square* for the quadratic function $f(x) = -4x^2 + x + 1$, and *do not skip any steps*. Write your answer in the form $f(x) = a(x - h)^2 + k$ for some values a , h , and k .

$f(x) =$ _____

7. Find the largest y -value of the function $f(x) = 1 - 4x - 2x^2$. (**Hint:** Vertex?)

Largest y -value: _____

8. *Completely simplify* the following compound fraction.

$$\frac{2}{1 - \frac{x-1}{x+1}}$$

Simplified fraction: _____

9. Solve the following equation for the variable x . (**Hint:** Use the one-to-one law for logarithms.)

$$\ln\left(\frac{2x+1}{4x-1}\right) = \ln\left(\frac{4x-7}{8x+3}\right)$$

$x =$ _____

10. *Completely simplify* the following compound fraction.

$$\frac{\left(\frac{a^{-1}bc}{a^3b^{-4}}\right)^{-1}}{\left(\frac{ab^{-1}c}{a^{-3}b^3}\right)^2}$$

Simplified fraction: _____

11. Factor the following expressions *completely*. (**Suggestion:** You might want to use the factoring rule $A^2 - B^2 = (A + B)(A - B)$.) *Put your work in the space provided below the problem.*

(a) $2x^5 - x^4 - 15x^3 =$ _____

(b) $x^2 - 16y^2 =$ _____

(c) $x^3 - x^2 - xy^2 + y^2 =$ _____ (**Hint:** Factor by grouping the first two terms together and then the last two terms together.)

12. For which value(s) of b does the following equation have *only one root*?

$$3x^2 + bx + 1 = 0$$

$b =$ _____ (list all the values separated by commas)

13. Solve the following equation for all values of x (if any) using the *quadratic formula*.

$$5x^2 - 15x + 11 = 0$$

$x =$ _____ (List all the values of x separated by commas. If there are no solutions, write 'None'.)

14. Solve the following equation by viewing it as a 'quadratic equation in disguise'. (**Hint:** Substitute $y = (5x - 1)^2$ and solve the resulting quadratic.)

$$4(5x - 1)^4 - (5x - 1)^2 - 3 = 0$$

$x =$ _____ (List all the values of x separated by commas. If there are no solutions, write 'None'.)

15. Consider the circle $x^2 + y^2 - 6x + 10y - 4 = 0$. What are its *center*, *radius*, and *circumference*? Recall the circumference of a circle is πd where d is the diameter.

Center: _____

Radius = _____

Circumference = _____

16. Write down the *equation* of a circle with center $(1, -2)$ and radius 5. Express your answer in the form $x^2 + y^2 + ax + by + c = 0$.

Equation: _____

17. Functions or not? Write 'T' if the equation represents a function and 'F' if it is not.

(a) _____ $y^5 = x^2 + x - 5$

(b) _____ $y = -3|x + 4| + 1$

(c) _____ $y^4 = x^8$

(d) _____ $x = 0$

(e) _____ $y^3 = \ln 9$

18. Write down the *domains* of each of the following functions. *In each case, express the answer as an interval or the union of intervals.*

(a) $f(x) = \log_2(x + 1)$; Domain: _____

(b) $f(x) = 4x^3 + x^2 + 7x - 8$; Domain: _____

(c) $f(x) = \frac{x^2+3}{x^2-4}$; Domain: _____

19. What is the *range* of the function $f(x) = -e^{x-6} + 13$? *Express the answer as an interval or the union of intervals.*

Range = _____

20. Let $f(x) = 2x^2 - x + 4$. *Compute and completely simplify* the difference quotient

$$\frac{f(x+h) - f(x)}{h}$$

for this function.

Difference quotient: _____

21. Consider the function $f(x) = -\ln(x - 1) + 2$. Using an ‘arrow diagram’, show how $f(x)$ is obtained using graph transformation from the base graph $\ln x$. *Make sure to explain the operation above the arrow.*

22. Suppose the domain of $y = f(x)$ is $[0, 7]$ and the range is $[-2, 4]$. What is the *domain* and *range* of the function $g(x) = 5f(2x - 9) + 13$? *Express the answers as intervals or the union of intervals.*

Domain: _____

Range: _____

23. One-to-one or not? Write ‘T’ if the function is one-to-one and ‘F’ if it is not.

(a) _____ $y = -4x + 7$

(b) _____ $y = x^8 + x^2 - 10$

(c) _____ $y = \ln(x^4 + 2)$

(d) _____ $y = \left(\frac{1}{2}\right)^x$

(e) _____ $y = \sqrt{5}$

24. Let $f(x) = 3x^2 - x - 4$ and $g(x) = 2x + 3$.

(a) Find and simplify the expression $f(2x) - 3g(x^2)$.

$f(2x) - 3g(x^2) =$ _____

33. **(CALCULATOR)** Suppose that \$20,000 is invested at 6.15% compounded *continuously*. What is the value of the investment after 5 years has elapsed?

Value of the investment (round to the nearest cent): _____

34. **(EXTRA CREDIT)** Strontium 90 is a radioactive material that decays according to the equation $A = A_0 e^{-0.0244t}$ where A_0 is the initial amount present (in grams), and A is the amount present after t days. Assume that $A_0 = 100$ grams.

(a) How much strontium 90 will be present after 9 days?

Amount remaining (round to the nearest tenth of a gram): _____

(b) What is the half-life of strontium 90?

Half-life of Iodine 131 (round to the nearest day): _____

35. **(EXTRA CREDIT)** Solve the following equation. *Do not use your calculator. Give exact values of x .*

$$3^x = 2^{x^3}$$

$x =$ _____ (separate multiple answers by commas)