2005

Week 2 Vectors and Newton's Laws

We are now going to explore **classical mechanics**, which applies to every realm of interaction save

(a) extremely small particles (atoms, sub-atomic particles),
(b) relativistic (speeds > $10^{-3}c$, speed of light), or
(c) cosmological scales (galaxies, dense matter)

Classical mechanics is embodied in Newton's Laws (**Sir Isaac Newton**, (1642-1727), shown at left.),

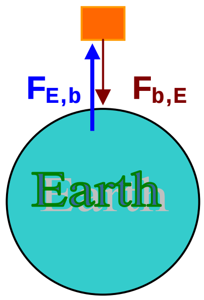
which was built on the work of other masters, Tycho Brahe, J. Kepler, G. Galileo, R. Descartes (the *Cartesian* set is named in his honor), among others. **Pre-Newtonian**: one set of rules applied to the heavenly bodies, another to earth-bound bodies. How else could wheels slow to a halt on Earth, while Jupiter continues its celestial orbit in the heavens? It is not unreasonable that the first naïve "guestimation" was that two sets of laws co-existed.

It took the meticulous observations of men like Brahe and Kepler, the critical doubt of scientists such as Copernicus which greatly enabled the brilliant deductive prowess scientists like Galileo and Newton to recognize that one set of physical law ruled the heavens and the Earth.

I. **Causes of motion** – ***interactions*** between bodies. Some are long range, such as attraction due to gravity, some are seemingly short range as the interaction between golf club and golf ball. The ball's trajectory is altered only by immediate contact with the club. Trying to mathematically describe these interactions remains the major pastime of theorists and computational scientists and the objective of most physics experiments.

Galileo Galilei surmised: Isolated forces do not exist, rather, forces occur in pairs. The two forces are acting on different objects (the two objects that are interacting). Thus to identify a force, we must be able to name the object acted upon as well as the object that acts upon it. For instance, all objects are attracted to the earth due to its gravitational pull, so we could label the force due to this interaction as $F_{b,E}$, the force on a body due to the Earth. But that's just ½ of the story. Newton's 3rd law assures us that there is an equal and opposite pull on the earth due to the object, amazing as it first seems!

In other words, there is pull on the object (**b**) due to the Earth, and an equal but opposite pull on the Earth due to the object. (See figure below). This pull is what we call a **force.**



More about Forces: Force is a derived quantity [F] = $\frac{ML}{t^2}$, the SI unit is Newton (N) $\equiv \frac{kg \cdot m}{s^2}$

To compare with US Customary Units, 1 N $\equiv$ 0.2248 lb

Force is a **vector** quantity, it has magnitude, sign, unit and **direction**. To compare,

Mass is a scalar quantity that has only magnitude and unit (SI: kg) , where as charge is a scalar quantity with magnitude, sign and unit (SI: Coulomb) It is not surprising that force has direction, the putt of a golf ball will make birdie or will miss the hole. The contact of ball with bat either produces a line drive, a fly ball, a foul, etc. Driving due east, one will not arrive at their home due south.

1. **Adding collinear forces (1-D):** If two forces are applied to an object in the same direction, (forces are parallel) the magnitude of the resultant force is the sum of the two magnitudes.

   Ex. 1: Two men push a heavy crate to the left. One supplies 200 N, the other man supplies 163 N. The net force force (magnitude and direction) applied to the crate is, **200 +163 = 363 N to the left**.

   If two forces point in the opposite direction (antiparallel), the magnitude of the resultant force is the difference of the two magnitudes. **Ex 2.** Suppose the two men above pushed in opposite directions, the 163 N applied to the left and the 200 N applied to the right. Let $\hat{l}$ be a unit vector that points to the left, $\hat{r}$ be a unit vector that points to the right (obviously $\hat{r}$= -$\hat{l}$). A unit vector has length 1 (unity). For now, think of the notation as a place holder, keeping different directions separate, so that we won't accidentally add them together.

   Then: F = (163 – 200) $\hat{l}$ = (200 – 163)$\hat{r} \Rightarrow$ **F = 37 $\hat{r}$ N [to the right]**.

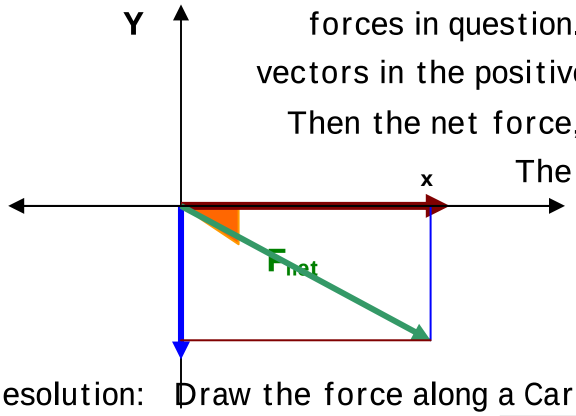
2. **Adding noncollinear forces (2D): Resolving components**

   In order to add forces that are not collinear, each force has to be **resolved** into its x and y components in a convenient reference frame. Suppose we have a 6 N force directed east and a 3 N force directed south. What is the net force, $\mathbf{F}_{net}$?

As east and south is already perpendicular to each other, one could align these vectors along a Cartesian set:

A. Graphical resolution: Graphically, the resultant force is just the diagonal of the rectangle formed by the two forces in question. Let $\hat{\mathbf{x}}$, $\hat{\mathbf{y}}$, and $\hat{\mathbf{z}}$ be unit vectors in the positive x, y, and z directions. Then the net force, $\mathbf{F}_{net}$: $(6\,\hat{\mathbf{x}} - 3\,\hat{\mathbf{y}})$ N

The net force has a south-easterly direction.



B. Algebraic resolution: Draw the force along a Cartesian set as before.

The **magnitude** of the net force is: $|\mathbf{F}_{net}| = \sqrt{6^2 + 3^2}$ N = **6.71 N**

The angle that the net force makes with the x-axis is determined via:

$\cos(\theta) = \dfrac{6.00}{6.71}$, thus $|\theta| = \cos^{-1}(\dfrac{6.00}{6.71})$ = **0.4642** radians or **-26.6°** [1] **(why minus?)**

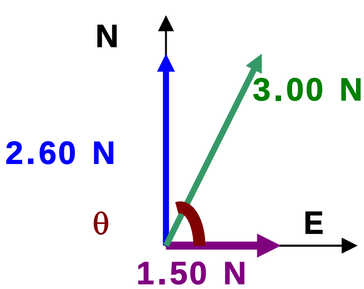
Convention: if tracing angle from x-axis to $\mathbf{F}_{net}$ is *counter*clockwise, the algebraic sign (θ) is positive; if the trace is *clock-wise*—as presently—sign(θ) is negative. In summary:

x-component of $\mathbf{F}_{net,x}$ = **6.71** × **cos(-26.6°)** = **6.71** × **cos(26.6°)** = **6.00 N**
y-component of $\mathbf{F}_{net,y}$ = **6.71** × **sin(-26.6°)** = **-6.71** × **sin(26.6°)** = **-3.00 N**

∴ The force can be written as $\mathbf{F}_{net}$ = **6.71[cos(26.6°)** $\hat{\mathbf{x}}$ **-sin (26.6°)** $\hat{\mathbf{y}}$**] N**

Alternatively, one could state force **is 6.71 N** directed **26.6° south of east.**

Ex. 2: Two horses pull a load, one horse applies a force of 2.60 N due north, the other applies 1.50 N due east. What is the magnitude of the net force and the angle (°) it makes with the x-axis, if due east taken to be the positive x axis?



Solution: $|\mathbf{F}_{net}| = \sqrt{2.60^2 + 1.50^2}$ N = **3.00 N**

The angle that $\vec{F}_{net}$ makes is found by setting

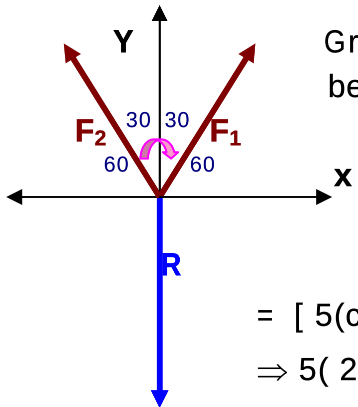
$\cos(\theta) = \dfrac{1.50}{3.00}$, thus $|\theta| = \cos^{-1}(0.50)$ = **60.0°** (or 1.05 radians).

Thus $\mathbf{F}_{net}$ = **3.00[cos(60.0°)** $\hat{\mathbf{x}}$ **+ sin(60.0 °)** $\hat{\mathbf{y}}$ **] N**

Alternatively, one could state that the force is **3.00 N,** directed **60° N of E.**

[1] (360° = 2π radian ; 1.7453 × 10^{-2} radian per degree)

Ex. 3 Suppose that two horses are tugging at a load, but the load isn't moving. Both horses exert a force of 5.0 N each but the angle between them is 60°. What must be the force of resistance (static friction) acting on the load?



Graphically, as the load isn't budging, the resultant force must be zero. So the resisting force must point as shown. By positioning the forces exerted by the horse symmetrically about the ***y***-axis, we see that the opposing force must be directed along the ***–y*** axis. Clearly the x-components cancel. We write $\mathbf{F}_{net} = 0 = \mathbf{F_1} + \mathbf{F_2} + \mathbf{R}$

$= [\ 5(\cos(60^o)\,\hat{\mathbf{x}} + \sin(60^o)\,\hat{\mathbf{y}}) + 5\,(-\cos(60^o)\,\hat{\mathbf{x}} + \sin(60^o)\,\hat{\mathbf{y}})]\text{N} + \mathbf{R}$

$\Rightarrow 5(\ 2\sin(60^o)\,\hat{\mathbf{y}})\ \text{N} + \mathbf{R} = 0$ Thus: $\mathbf{R} = -10\bullet(\frac{\sqrt{3}}{2})\,\hat{\mathbf{y}} = \mathbf{-(17.3\ N)}\,\hat{\mathbf{y}}$

Resolved in this coordinate system, the resisting force is 17.3 N along the ***–y*** axis.

II. Newton's Laws of Motion:

In 1687, *Philosophiae Naturalis Principia Mathematica* was published by Sir Isaac Newton. The *Principia* states 3 laws of motion that is the foundation of classical mechanics:

1. **Law of Inertia**—Inertia is the tendency of an object to resist motion; simplistically this quantity is proportional to the mass of the object. Newton's 1st law states: If no forces act on an object, then its speed and its direction does not change. Alternatively: If the object is at rest, it remains so with speed zero; if it is moving, it moves at a constant speed in a straight line.

   Aristotle—the natural state of an object was at rest. To move a force had to continuously be acting on it.

   Galileo—by *imagining* a frictionless surface, he foresaw the natural state of a body is to remain at constant speed. (Rolled balls up inclined planes).

2. **Acceleration** (a vector quantity) arises when a non-zero net force acts on an object of mass m according to $\vec{F} = m\vec{a}$

   When $\vec{F} = 0$, no net forces act on the object; it does not accelerate and its velocity is constant. The object is in ***translational equilibrium***. Further, if it is at rest, we say that the object is in **static equilibrium**, if it is moving at a constant speed, it is at **dynamic equilibrium**.

3. **Forces are Paired:** In an interaction between two objects, the force that each exerts on the other is equal in magnitude and opposite in direction.

A. **No object exerts a force without experiencing a force** on another body. It is sometimes remarked that there are no isolated forces.

When a ball (previously tossed into the air) is pulled down-ward due to Earth's attraction, the Earth is pulled upward w/ a force of the same magnitude due to the ball's attraction. Why do we not see this drastic effect? B/c Earth is *so much more massive* than the ball.

Ex. A ball weighs 0.30 kg. The Earth weighs about 6.0 x 10^{24} kg. It's acceleration towards the Earth is 9.81 $\frac{m}{s^2}$. Estimate the acceleration of the Earth caused by the ball.

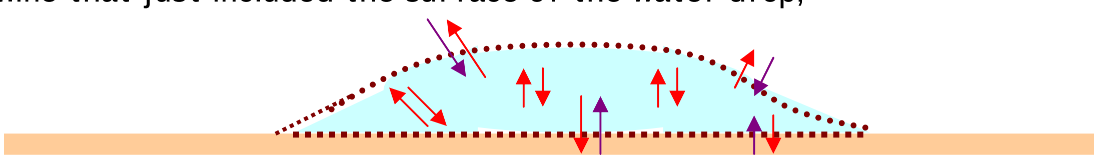
Solution. Let $\mathbf{F}_{b,E}$ be the force on the ball due to the Earth, then $\mathbf{F}_{E,b}$ is the force on the Earth due to the ball. By Newton's 3rd law, $\mathbf{F}_{b,E} = -\mathbf{F}_{E,b}$ and by Newton's 2nd law, $\mathbf{F} = m\mathbf{a}$, thus, for the ball:

0.30 kg (9.81 $\frac{m}{s^2}$) = (6.4 x 10^{24} kg)A_E, where A_E is the acceleration of the Earth due to the ball. Thus, $A_E = \frac{0.30}{6.4\times10^{24}}(9.81\ \frac{m}{s^2}) = \mathbf{4.7 \times 10^{-25}\ \frac{m}{s^2}}$

a pathetic acceleration indeed! What do we mean by pathetic? Well, a hydrogen atom has width $\approx 10^{-10}$ m. Thus, with this acceleration, in 20 seconds, the Earth has moved a distance 10^{-13} , barely $\frac{1}{1000}$ the diameter of a hydrogen atom!

B. **Internal and External Forces:** Suppose that we are interested in analyzing the forces acting on a drop of water on the windowsill.

The drop of water (0.1 mL) consists of about 3 x 10^{21} water molecules. These molecules interact with each other—according to the 3rd law—as well as with the molecules lining the window sill and particles in the surrounding air. If we drew an outline that just included the surface of the water drop,



all the 3rd law pairs that fall within our boundary cancel each other. The forces that fall within our boundary are called **internal forces**. However, some of the mutual interactions between water and air, water and sill will not fall within the boundary drawn. Thus some of those forces will not cancel. The forces that act on the raindrop, but fall outside our boundary are called **external forces**.

The point of all this is that the application of Newton's 2nd law to find the acceleration of the object requires the vector sum of only the external forces (the appropriate force-"member" of the interaction pair). Can you identify the internal and external forces in the diagram above?

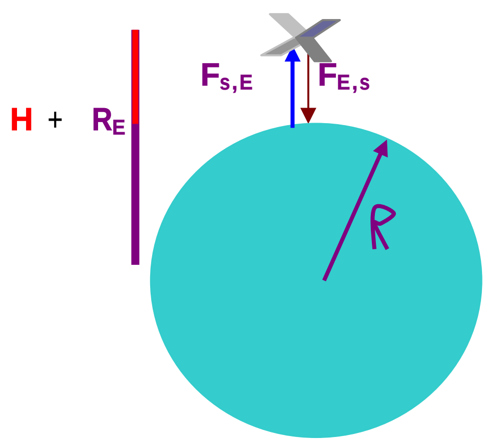
C. **Inertial Reference Frame required:** Notice, the 2nd and 3rd laws of Newton are valid only as long as an inertial reference frame (a reference frame in which the law of inertia holds, is used to record the motion.

D**.Gravitational Force—Law of Universal gravitation:** Newton proposed the following mathematical form of the 2 body interaction between two bodies **F =** $\frac{GMm}{R^2}$ where F is the force between two bodies, M and m are their masses, R is the distance between their centers, and G is a constant, termed the **universal gravitational constant**,

$$\mathbf{G} = 6.7 \times 10^{-11} \frac{m^3}{kg \cdot s^2}$$

One of the triumphs of Newton's formula was that it entirely reconciled the observations of Johannes Kepler (1571-1630), namely that the period squared of a planet's orb around the sun was proportional to its radius cubed.

1. **Earth bound bodies:** For objects whose height above Earth's surface is much smaller than Earth's radius, this force due to gravitation can be greatly simplified, using an approximation arising from a Taylor expansion similar to what was exhibited during last week's lectures.



Let the satellite be height **H** above Earth's surface. That mean's it's a distance **H**+**R**$_E$ from the Earth's center, thus, the mutual force of gravitation is: **F =** $\frac{GMm}{R^2}$ =

$$\frac{GMm}{(R_E + H)^2} = \frac{GMm}{R_E{}^2} \frac{1}{(1+\frac{H}{R_E})^2}$$

$$m\left(\frac{GM_E}{R_E{}^2}\right)\left(1 - 2\frac{H}{R_E} + \ldots\right) \text{ very nearly } \approx m\left(\frac{GM_E}{R_E{}^2}\right) = mg,$$

where **g**, the *local* acceleration due to gravity ≡ $\frac{GM_E}{R_E{}^2}$ **= 9.81** $\frac{m}{s^2}$**.** Thus the **weight** of an object possessing mass **m** on Earth (or very near its surface), **W = mg.**

Question: What would be the local acceleration due to gravity near the surface of the Moon? $M_{Moon} = 7.35 \times 10^{22}$ kg, $R_{Moon} = 1.74 \times 10^{6}$ m

III. Mechanical Forces—Next, there are classes of forces that we will encounter again and again in our classical mechanics problems. Contact forces, tension and the spring fore are among these. The spring force has been considered previously.

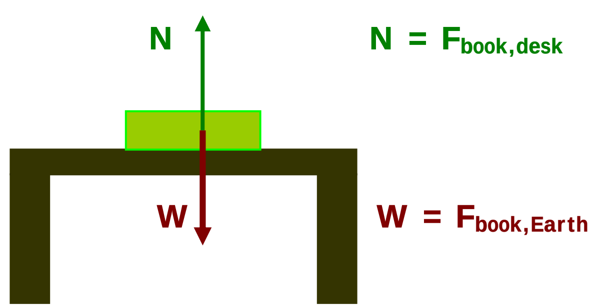
1. Contact Forces

Contact force results from the electromagnetic (mostly electrostatic) interaction between many atoms in the objects that are in contact with each other (share a common surface). Though electromagnetic interaction between a pair of charges tend to be long range, when there are 10^{20} –10^{30} such charges, an effect called screening—near cancellation of positive and negative charge—cancels virtually all of these electrostatic interaction except at the surface of contact. We will consider two of these:

2. Normal—so called because its direction is always perpendicular to the surface of contact. Why does a book rest on a desk? The net force must be zero, but we know that the book pushes downward on desk with a force equal to its weight (due to attractive pull from Earth).

$$\mathbf{F}_{desk,book} = \mathbf{W}_{book} \text{ (directed downward)}$$

Why doesn't the book crash into the desk? B/c of Newton's 3rd law, The desk exerts an upward force of equal magnitude on the book,



3. Friction—a contact force *parallel* to the surfaces. Denote by lower case **f**.

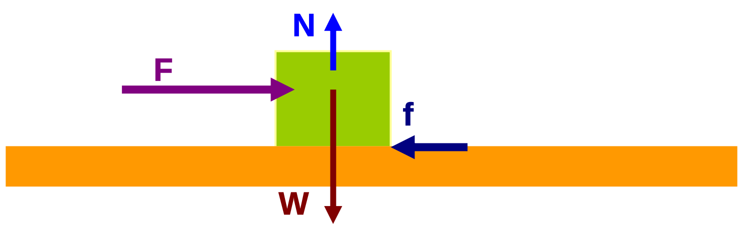
a. Static friction—a child reposed at top of slide before the push off; a crate sitting on the floor $\mathbf{f}_{[MAX]} = \mu_s \mathbf{N}$ where μ_s is *the coefficient of static friction* and **N** is the normal force. Notice that the *maximum friction* to resist motion is given by $\mu_s N$. In other words, depending upon the given static situation,

$$\mathbf{0} \le \mathbf{f} \le \mu_s \mathbf{N}$$

Thus, in a problem involving an object that is at rest, we cannot apriori assume that $f = \mu_s N$. The static friction works in the direction that resists the onset of sliding.

b. Kinetic friction—occurs when a child slides down a slide, or a crate is dragged across a floor. If the object is in motion, the expression for friction becomes $\mathbf{f} = \mu_k \mathbf{N}$ where μ_k is *the coefficient of kinetic friction*. Once the body starts moving, the resistance between surfaces is decreased somewhat. This is manifested by the coeff. of kinetic friction being less than the coefficient of static friction: $\mu_k < \mu_s$. Kinetic friction works in direction that opposes sliding, once it starts (obviously, in opposite direction of motion).

Ex. In order to push an article of furniture (weight 750 N) across the floor, a force of 450 N must be applied. (a) Find the coefficient of kinetic friction. (b) If one applies only 110 N, the furniture doesn't budge. Assuming that $\mu_s = 1.2\mu_k$, find the force due to friction.



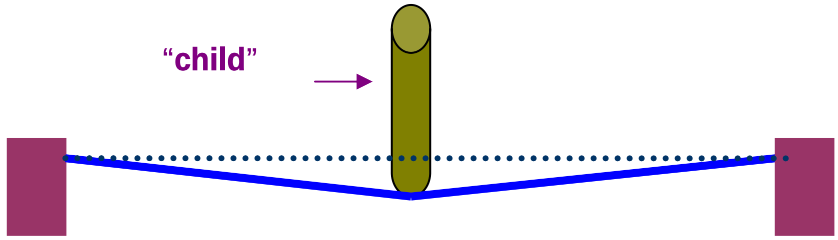
(a) $f = \mu_k N$, so $450\text{ N} = \mu_k(750\text{ N})$, $\mu_k =$ **0.60**

(b) $\mu_s = 1.2\mu_k$ implies $\mu_s = 0.72$, but ***<u>don't be misled</u>***. It's tempting to write $\mathbf{f} = \mu_s \mathbf{N}$, as in the kinetic case. However, as the furniture does not move though 110 N is applied, a force of at least 110 N is opposing it. Thus, the frictional force **f** need only be **110 N**.

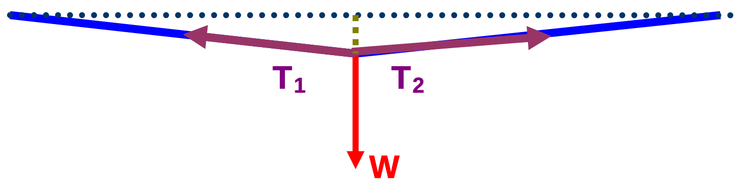
Another important force is the tension.

4. **Tension—**a flexible rope, cord, cable, etc. that is under tension exerts force on an object at both ends called **tension**. Unlike a spring, the change in length of the rope is almost negligible and the rope can only pull, as it can virtually not be compressed, it cannot "push" back.

Ex. A child of weight 40 N stands in the middle of a tight rope placed close to the mat for safety. The rope, with equilibrium length 2.50 m, is distended 6 cm from its equilibrium position. What is the tension in the tight rope?



A simplified, free body diagram, FBD, shows:

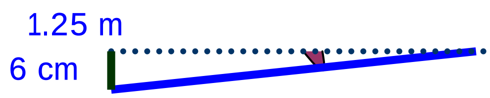


Because the child is not accelerating, the net force must be zero:

$\mathbf{F}_{net}$ **= ma =** $\mathbf{W} + \mathbf{T}_1 + \mathbf{T}_2 = 0$. (Clearly, and, appealing to symmetry, $|\mathbf{T}_1| = |\mathbf{T}_2| = \mathbf{T}$). In terms of the x and y directions:

$-\mathbf{T}_{1,x} + \mathbf{T}_{2,x} = 0$ (**W** possesses no horizontal component)

$-\mathbf{W} + \mathbf{T}_{1,y} + \mathbf{T}_{2,y} = 0$, or $\mathbf{W} = 2\mathbf{T}_y$ forgoing the 1 & 2 subscripts.



From diagram above, $\mathbf{T}_y = \mathbf{T}\sin\theta$. Thus $\mathbf{T} = \mathbf{W} \div 2\sin\theta$. We've only to find the angle[2] which is given by: $\tan^{-1}(\frac{0.06\text{ m}}{1.25\text{ m}}) = 2.75^o$, so that tension $T = 40\text{ N} \div (2\sin(2.75^o)) \sim$ <u>417 N</u>.

IV. Unification—one of the quests of physics is to try to account for the many types of forces encountered in the universe via a set that contains the fewest possible, and therefore *fundamental interaction*. Great progress had been made and its is now thought that all the forces can be accounted for by just four fundamental interactions:

Gravitational—long range, always attractive, weakest of the four.
Electromagnetic—wide range of influence, weak to medium strength
Strong and weak nuclear forces—shortest ranges; strong nuclear forces are among the strongest known. [GeV-TeV, 10^{-15} m], the weak force range is shortest of all: 10^{-17} m (describes certain radiation, neutrinos, etc.; without weak thermonuclear interactions in Sun, sunlight wouldn't be produced)

Newton's *Principia* brought a certain level of unification, soundly demonstrating that the laws governing the motion of the planets was the same as that of an apple falling to Earth. An ongoing goal of theorists is to find the one formulation of physical law that account for all the fundamental interactions.

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2 Or, it is convenient when θ is suspected to be very small to use radians): $\tan\theta = \frac{0.06\text{ m}}{1.25\text{ m}} \approx 4.8 \times 10^{-3}$ in radians $\approx \theta$ also approximates $\sin\theta$ (θ very small), $\mathbf{T} = 40\text{ N} \div (2 \times 0.048) = 4.14 \times 10^2$ N