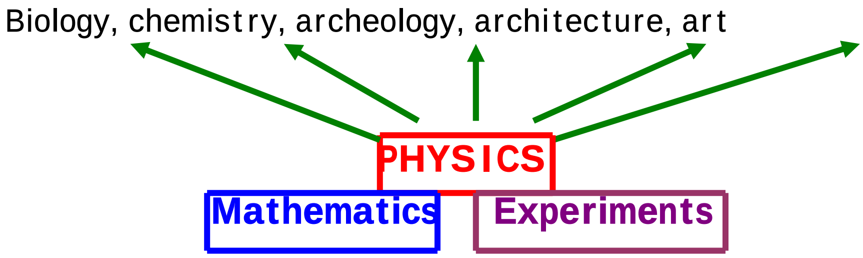
Week 1 lecture – Visits a few tools of the "Toolbox" for solving physics problems

Physics describes matter, energy, space and time at the most fundamental level. The robustness of its laws and principles are most often cast in the eloquent language of mathematics.



I. Math: Must be comfortable with

i. algebra, trigonometry, geometry
ii. interpreting graphs
iii. approximations and order of magnitude estimations
iv. translating English into math

Examples of math concept "tools":

1. factor—actually a ratio

"rate increased by a factor of 3" how do we translate algebraically?

Let R = 'previous' rate, R' = new rate, then **R' = 3R** equiv: $\frac{R'}{R} = 3$

2. percent increase/decrease $(1 \pm \frac{n}{\mathbf{100}})$

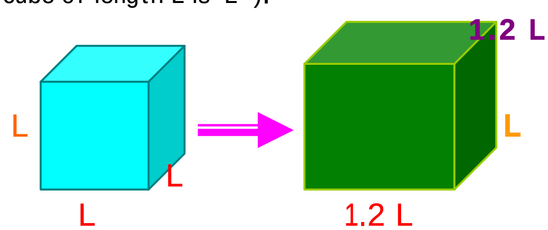
"population increase by 5%" how do we translate algebraically?

Let P_o = 'previous' pop'n, P' = new population, then $P' = P(1 + \frac{5}{100}) = \mathbf{1.05 \times P}$

Similarly, a 13 % decrease in temperature would imply—with T' being temperature after decrease $P' = P(1 + \frac{5}{100}) = \mathbf{1.05 \times P}$

Examples:

(1) Suppose that the length of each dimension of a cube increased by 20%? (a) By what factor did each dimension increase? (b) relate the new volume to the previous one. (The volume of a cube of length L is L^3).



(a) Solution: **let L' be the new length. 20% increase means that** $\mathbf{L' = L(1 + \frac{20}{100}) = 1.2 \times L}$

(b) $V' = (1.2 \times L)(1.2 \times L)(1.2 \times L) = (1.2)^3 \times L^3 = (1.2)^3 \times V \cong \mathbf{1.7 \times V}$.

(2) With *d* representing the diameter of a circle, find the new area when the diameter of a circle is doubled. (Area = $\frac{\pi}{4}d^2$) **Solution:** if A = previous area and A' = new area, A' = $\frac{\pi}{4}(2d)^2 = (2)^2 \times (\frac{\pi}{4}d^2) = 4A$ ∴ area <u>quadruples</u>.

II. **Empirical Considerations**

Physics is an empirical science. No matter how mathematically elegant its formulation, the predicted behavior must be borne by experiment. Some useful tools in this regard:

1. **Scientific Notation** –helps us to handle unwieldy large or small numbers. A number in scient. not'n is written as $C \times 10^n$ where

$$10 \geq C \geq 1$$

Ex 1: Radius of Earth: **6,380,000 m**. If the number is greater than unity (1), then the power of the exponent of 10 is the (# of digits – 1); or one can count by 1 the digits from right to left starting with 0: $\frac{6\ 5\,4\ 3\ 2\,1\,0}{6{,}380{,}000}$ arrive at

$\mathbf{6.38 \times 10^6\ m}$

Ex 2: Radius of hydrogen atom (H-atom): **0.000 000 000 053 m**

Count out to 1st non-zero digit, this is the (negative) power of 10

Thus we write radius of H-atom as $\mathbf{5.3 \times 10^{-11}\ m}$

2. **Significant figures**—as physics is an experimental science, the measurements made are subject to error, statistical or other randomness. Thus, the precision of the experiment must be reported. This clearly conveys to the scientific community the confidence that the experimentalist has in the result and thus the credibility of the experiment. For our purposes, *the experimentally reproducible place values in a figure are the significant figures*. For instance, if a volume is reported as 50.12 mL, the outcome of a specific interaction, the experimentalist is claiming that any other careful scientist will get 50.12*xxx* mL if they observe that same interaction. In other words, the 1st four digits they measure should be 5, 0, 1, and 2. Any differences might first appear in the 5th, etc. place. Obviously, then, precision depends upon the resolution of the experimental set-up.

How the number of significant figures is determined: Summary

1. non-zero digits are significant *43618 contains **5** sig figs*.
2. zeros trailing the decimal are significant
   *32**.**0000 contains **6** sig figs, the number is precise to the 10 thousandth place.*
3. zeros that are merely place-holders are not significant
   *0.000 000 000 053 contains only **2** sig figs.*
4. zeros between nonzero digits (*captive* zeros) are significant.*207 contain **3** sig figs*.

Mathematical operations involving significant figures- When the quantity sought after is a composite of two or more measurements, the significant figures of the composite result must reflect that of the least precise measurement. This plays out a little differently for addition/subtraction vs. multiplication/quotients, so we examine each case:

Addition and subtraction. Suppose that one wants to determine the volume of a beaker of water into which an ingot of copper has been placed. The volume of the water is 84.07 mL, and the volume of the copper was determined by displacement technique to be 7.1389 mL. What is the volume of copper + water?

Naïvely, just using calculator : V = 84.07 mL + 7.1389 mL = 91.2089.

However, as volume of water is only known to 100th's place, we round off to 100th 's place 91.20|89 ≅ 91.2*1* Thus **91.21** mL is reported,

In general, *the sum or difference of numbers must be rounded to have precision of the least precise term in the sum or difference*.

Multiplication/division. The density of a piece of copper is sought. The mass of a 0.500×10^{-6} m^3 sample is 4.4 gram. What is the density of copper in kg/m^3 ? Density = Mass ÷Volume, so

$$D = \frac{4.4 \text{ g}}{5.00 \times 10^{-7} \text{ m}^3} = 8.8 \times 10^{+6} \frac{\text{g}}{\text{m}^3} \times \frac{10^{-3}\text{kg}}{\text{g}} = 8.8 \times 10^{+3} \frac{\text{kg}}{\text{m}^3}$$

Notice that even though volume was known to **3** sig figs, density is known to only **2**, thus result is reported to only **2** sig figs. The actual density of copper is $8.94 \times 10^3 \frac{\text{kg}}{\text{m}^3}$

In general, *the product or quotient of experimental numbers is reported with the same number of sig figs as the number with the fewest sig figs.*

(Note also the conversion factor that facilitated conversion from g to kg.)

Finally

Exact relationships do not affect number of significant figures. There are exactly 12 inches in one foot, 100 cm ≡ 1 m, 2.54 cm ≡ 1 in, 8.64×10^4 s ≡ 1 day, etc.

The fact that 12 contain **2** digits doesn't limit the precision of a complex quotient that involves the conversion of, say, feet to inches, to **2** sig figs.

3. UNITS—b/c physics is an empirical science, the reports of all observations made must clarify the units used to measure the physical quantity. A **unit** is a determinate quantity (of mass, length, time, temperature, and charge) adopted as a standard of measurement.

Ex: If a friend owes you 5, it matters whether he owes you 5 cent, 5 dollar, or 5 grand! If the volume of a liquid is reported as 230, is that 230 mL or 230 gallon? In other words, reporting a number with out a unit has no meaning, and will contribute to the loss of points on graded assignments. Other labs cannot verify or invalidate the claim.

As alluded to before, the fundamental quantities are: of mass m, length L or *l*, time *t*, temperature T, and charge Q. All other quantities are derived: *viz*, speed = length/time,

density = mass/ volume = $\frac{m}{V} = \frac{m}{l^3}$

In physics, the units adopted by most physicists adhere to the Système International d'Unités [SI], a *metric* system (that is, based upon powers of 10).

<u>Short list:</u>

Mass - kilogram (kg)
Length – meter (m)
Time - second (s)
Temperature - Kelvin K
Charge - Coulomb C

In the US, Customary Units are unfortunately still employed, (although SI is finding its way into the grocery store and records of temperature): pound (weight, not mass), foot, and seconds. In US customary units, mass is a derived quantity.

A. Prefixes, very useful to handle smaller or larger quantities than the amount indicated by a SI unit. In other words, quantities other than SI might be more useful for analyzing your line of work.

PREFIX	**abbrev**	**10^n**
exa	E	10^{18}
peta	P	10^{15}
tera	T	10^{12}
giga	G	10^{9}
mega	M	10^{6}
kilo	k	10^{3}
centi	c	10^{-2}

milli	m	10^{-3}
micro	μ	10^{-6}
nano	n	10^{-9}
pico	p	10^{-12}
femto	f	10^{-15}
atto	a	10^{-18}
zepto	z	10^{-21}

Example *fm* (femtometer) typical influential distance of nuclear "strong force".
Ps (picosecond) typical relaxation time of a hydrogenic vibrational mode.

Different fields adopt different local conventions.

***Nuclear physics*: Energy: electron volts (eV) instead of the SI derived unit Joule (1 eV ≅ 100 kJ)**

Chemistry: **Volume liter (L) in stead of m^3 ;**
Pressure: atmosphere or mm Hg instead of the SI derived unit pascal,
Energy: kilojoule or calorie instead of the SI derived unit Joule

B. Conversions—At times two different sets of units are implied in a problem, so we will have occasion to convert from one set of units to another. Easiest to accommodate the conversion factor as a ratio between the units to be exchanged. (obviously these units must describe the same quantity: speed, density, mass, volume , money, etc.)

Example:Problem 19, pg. 20 The density of mercury is 1.36×10^4 kg/m^3 . Convert to g/cm^3.

Solution, must convert mass unit and volume unit. 1 kg = 10^3 g,
1 m = 10^2 cm, thus, cubing 1 m^3 = 10^6 cm^3 Now treating these relations as conversion factors: $D = 1.36 \times 10^4 \frac{\text{kg}}{\text{m}^3} \times \frac{\mathbf{10^{-3}kg}}{\mathbf{g}} \times \frac{1\,\text{m}^3}{\mathbf{10^6\ cm^3}} = 13.6 \frac{\text{g}}{\text{cm}^3}$

(5). Dimensional Analysis—Dimensions are basic types of units, such as mass (M), length (L) and time (t). Quantities can be added or subtracted only if they have the same dimensions. Analyzing a problem by looking at the dimensions involved may lead to the answer (within a scaling factor of ½, π, etc) in many cases.

Example: Problem 31. Frequency $f \sim s^{-1}$, force constant, k, of a spring has units g/s^2. (The force constant directly relates to the stiffness of the spring). A *massing chair* is used to determine the mass of objects on a space shuttle. The chair (mass 10 kg) is attached to a vertically hanging spring and the object which mass is to be determined is placed in the chair. The frequency at which the spring oscillates is related to the total mass of the load and the stiffness of the spring. If the frequency of oscillation is 0.50 s^{-1} for a 62 kg astronaut, what would be the frequency of the oscillation for a 75 kg astronaut?

Solution: The quantities listed are:

Quantity:	**frequency**	**mass**	**force constant**
Dimensions:	1/t	M	$M/t^2 = Mt^{-2}$

As frequency is the response to the strength of the spring and the mass of the load, we can write frequency as a function of these quantities: $f \sim f(M, k)$.
Next, what combination of mass and force constant results in dimension of 1 over time (t^{-1}) ? Answer $\sqrt{\frac{Mt^{-2}}{M}} = \frac{1}{t}$ Thus, within a constant, at most,

frequency ~ $\sqrt{\frac{k}{M}}$ we now know how frequency depends upon M and k.

Unfortunately, k is not given. However, k's numerical value is not needed: we may solve our problem by invoking ratios. Let

f_1 correspond to M_1 = (62+10) kg = 72 kg in the 1st instance
f_2 correspond to M_2 = (75+10) kg = 85 kg in the next

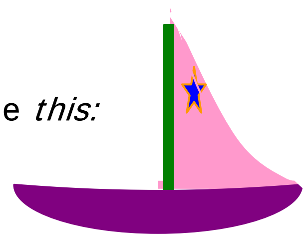
Then $\frac{f_2}{f_1} = \sqrt{\frac{k/M_2}{k/M_1}} = \sqrt{\frac{M_1}{M_2}} = \sqrt{\frac{72}{85}} \sim 0.912$ note how k's dependence dropped out.

Thus, $f_2 = 0.912 \times f_1 = 0.912 \times 0.50\ s^{-1} = \mathbf{0.46\ s^{-1}}$; the frequency of more massive astronaut is somewhat smaller. Notice how the subscripts are reversed—we say that frequency depends inversely upon the square root of mass. [By the way, the final set up in this problem is very similar to problem 5, pg. 19 of your homework.]

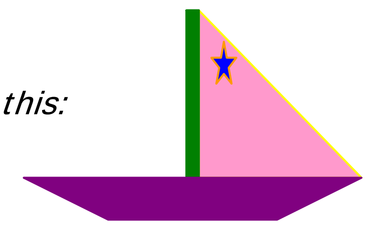
6. <u>Approximations</u>

1. Measurements are approximate—b/c of
   a. the limiting resolution of device used to make measurement
   b. the difficulty or technical inaccessibility of the measurement
      (measure the surface magnetic moment of a black hole)
2. Models are approximate—to solve problems in physics, we often analyze the real world via a simpler model. Otherwise, if every interaction was taken into account, the problem would be formidable, even impossible to solve.
   If 15% or so error could be tolerated, we'd have license to

approximate *this:*



by *this:*



As in a model, certain interactions are ignored, a valuable skill is to be able to recognize under what conditions do we expect the model to reasonably mimic the real interaction, and under what conditions it is not reasonable. For instance, why could we ignore the breeze's effects in estimating the time of fall of a stone from our hand to the earth, but not of a feather's release ?

(write on board☺) Model: simplifies but we must recognize the limit of its applicability

Another example: we sometimes can substitute a simpler mathematcal expression to calculate a value. For instance, if a number is very nearly unity (1), and its square root is sought, a binomial approximation may do well for the # of sig figs acceptable.

Let a number be written as

$1 \pm x$ If $x \ll 1$, then, the Taylor expansions of the following functions yield:

square root of almost 1 : $\sqrt{(1 \pm x)}$ ~ $1 \pm \frac{1}{2}x + \frac{1}{4}x^2 \pm$ etc. ~ $1 \pm \frac{1}{2}\boldsymbol{x}$

inverse of almost 1 : $\frac{1}{1 \pm x}$ ~ $1 \mp x + x^2$ Log of almost 1 : $\mathrm{Ln}(1 \pm x) \sim \pm x - \frac{1}{2}x^2$

Ex. Find (a) $\sqrt{1.006}$ (b) $(0.9948)^{-1}$ (c) Ln(0.9963)

Ans. (a) here x = 0.006, so $\sqrt{1.006} = 1 + \frac{1}{2}(0.006) = 1.003$

(b) 0.9948 = 1 – 0.0052 , thus answer is 1 + 0.0052 = **1.0052**

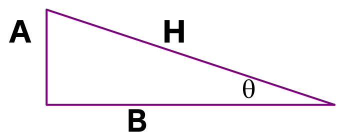
(c) -3.7×10^{-3}

7. Trigonometry—Velocities and forces are directional, therefore they are vectors, with a particular orientation in our reference frame. To see how much of the vector acts in a direction of interest will require resolving it into components along that direction. Component resolution requires *trigonometric relationships*. For example scenarios described as static equilibrium, motion along an inclined plane, or 2-D scattering of two bodies will call for resolving force/torque, force, or momentum vector components, respectively.

For a right triangle,

Pythagorean 's theory: $H^2 = A^2 + B^2$ thus $H = |H| = \sqrt{A^2 + B^2}$

H = hypothenuse |H| is magnitude of H



$A = H\sin\theta$

$B = H\cos\theta$

$A/B = \tan\theta$

Example. A force F possesses component 2.5 N in the *x* direction and 6.0 N in the *y* direction.

(a) Determine the magnitude of the force

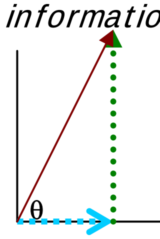
(b) Determine the angle in degrees that this force vector makes with respect to the x axis. (x ↔ due East). Use language S of E or N of E as appropriate.

Solution: $F = F_x\hat{x} + F_y\hat{y} = (2.5\text{ N})\hat{x} + (6.0\text{ N})\hat{y}$ where $\hat{x}$ and $\hat{y}$ are unit vectors,

(i) magnitude = length 1, thus, name "unit" (ii) orthogonal to each ⊥ to each other)

Unit vectors are an extremely useful way to *organize directional information*.

The above representation may be read " the force in newtons is directed 2.5 units east and 6.0 units north ".



(a) using Pythagorean's theorem $F = \sqrt{F_x^2 + F_y^2} = 6.5\text{ N}$

(b) to obtain the angle, we have $F_y/F_x = \tan\theta = 6.0/2.5 = 2.4$

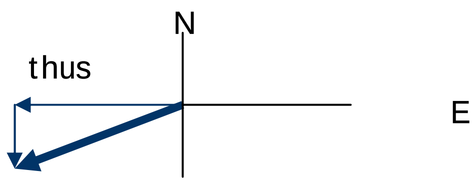
But we do not seek tanθ, but θ itself. Thus taking INVERSE tanθ (2nd function tan on many calculators) we obtain: θ **= $\tan^{-1}$(2.4)** = **<u>67°</u> N of E**

Example. A head-wind with speed 40.0 mi/hr is directed 30° S of W (4th quadrant). What is the speed in the south direction? In the west direction?

Solution: West ⇔ -x and South ⇔ -y thus

$V_x = -(40.0\text{ mi/h})\cos(30^\circ) = -34.6\text{ m/h}$

$V_y = -(40.0\text{ mi/h})\sin(30^\circ) = -20.0\text{ mi/h}$



Example. Vector $\vec{A}$ possesses magnitude 5.66 cm and $\vec{B}$ possesses magnitude 10.00 cm. The angles with respect to the positive x axis are, respectively,

angle($\vec{A}$) = 45.00° angle($\vec{B}$) = 36.87°

(a) Find the x and y components of the sum of these vectors, $\vec{C} = \vec{A} + \vec{B}$

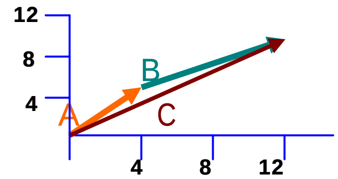
(b) Find the magnitude of $\vec{C}$ and the angle that $\vec{C}$ makes with the positive x axis (due East).

(a): $C_x = A_x + B_x = A\cos\theta_A + B\cos\theta_B =$ 5.66cos(45.00) + 10.00cos(36.87) **= <u>12.00 cm</u>**

$C_y = A_y + B_y = A\sin\theta_A + B\sin\theta_B =$ 5.66sin(45.00) + 10.00sin(36.87) = **<u>10.00 cm</u>**

(b) the magnitude of $\vec{C} \equiv |\vec{C}| = \sqrt{(10)^2 + (12)^2}$ = **15.62 cm**

the angle that $\vec{C}$ makes with the positive x , θ_C = $\tan^{-1}$(opp/adj) = $\tan^{-1}$(10/12) = **<u>39.80º N of E</u>**



8. <u>Graphing</u>—a visual aid that enables us to see patterns in the various relationships between two variables. It is the mathematical equivalent of the adage "a picture is worth a thousand words".

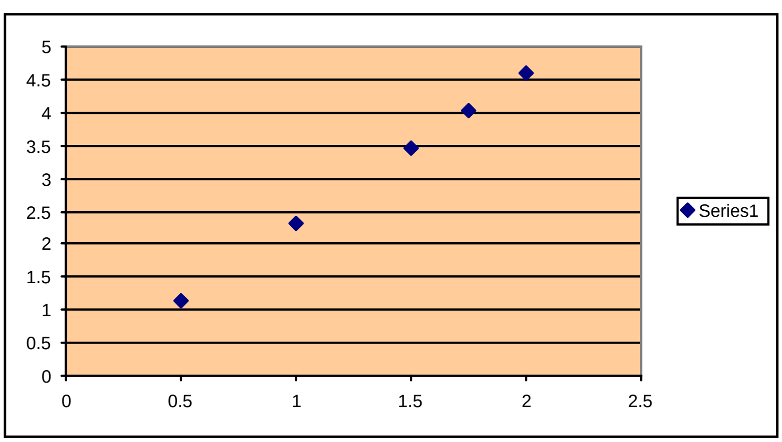
Suppose we wanted to see the relationship between the (slight) distension of a spring from its equilibrium length to the responding force of the spring. The displacement is the independent variable, as the experimentalist controls this change. The responding force of the spring (which, by Newton's 3rd law is equal and opposite to the force experienced by the spring) is the *dependent variable*, as it's the responding variable.

We might collect data in a table like this:

Displacement *x* (cı	**Force (*newton*, N ≡ $\frac{kgm}{s^2}$)**
0.50	1.14
1.00	2.30
1.50	3.45
1.75	4.02
2.00	4.59

However, a plot (graph) is much more revealing:

Displacement *x* in centimeters



We see that the force is nearly linear in displacement—in fact, this is the embodiment of **Hooke's Law** (for small displacements): $F = -kx$